Appendix

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On the Amount of Electricity which Flows through the Cross-Section of the Circuit in Galvanic Currents*)

[Translated by Susan P. Johnson and edited by Laurence Hecht]

A prefatory note from Kohlrausch says that the publisher desired for the Annals a report on work carried out jointly by Weber and Kohlrausch, whose results were presented in a more fundamental and conclusive way by Weber in vol. V of the treatises of the Royal Saxon Scientific Society in Leipzig, under the title *Elektrodynamische Maassbestimmungen, insbesondere Zurueckfuehrung der Stroemintensitaetsmessungen auf mechanisches Maass*, Leipzig, S. Hirzel, 1856. "Herewith I give a short precis".

1. Problem

The comparison of the effects of a closed galvanic circuit with the effects of the discharge-current of a collection of free electricity, has led to the assumption, that these effects proceed from a *movement of electricity* in the circuit. We imagine that in the bodies constituting the circuit, their *neutral* electricity is in motion, in the manner that their entire positive component pushes around in the one direction in closed, continuous circles, the negative in the opposite direction. The fact that an accumulation of electricity never occurs by means of this motion, requires the assumption, that the same amount of electricity flows through each cross-section in the same time-interval.

It has been found suitable to make the *magnitude of the flow*, the so-called *current intensity*, proportional to the amount of electricity which goes through the cross-section of the circuit in the same time-interval. If, therefore, a certain current intensity is to be expressed by a *number*, it must be stated, which current intensity is to serve as the measure, i.e., which magnitude of flow will be designated as 1.

*) *Poggendorf's Annalen*, vol. XCIX, pp. 10-25.

Here it would be simplest, as in general regarding such flows, to designate as 1 that magnitude of flow which arises, when in the time-unit the unit of flow goes through the cross-section, thus defining the measure of current intensity from its *cause*. The unit of electrical fluid is determined in electrostatics by means of the *force*, with which the free electricities act on each other at a distance. If one imagines two equal amounts of electricity of the same kind concentrated at two points, whose distance is the unit of length, and if the force with which they act on each other repulsively, is equal to the *unit of force*, then the amount of electricity found in each of the two points is the measure or the unit of free electricity.

In so doing, *that force* is assumed as the unit of force, *through which the unit of mass is accelerated around the unit of length during the unit of time*. According to the principles of mechanics, by establishing the units of length, time, and mass, the measure for the force is therefore given, and by joining to the latter the measure for free electricity, we have at the same time a measure for the current intensity.

This measure, which will be called the *mechanical measure* of current intensity, thus *sets as the unit, the intensity of those currents which arise when, in the unit of time, the unit of free positive electricity flows in the one direction, an equal amount of negative electricity in the opposite direction, through that cross-section of the circuit*.

Now, according to this measure, we cannot carry out the measurement of an existing current, for we know neither the amount of neutral electrical fluid which is present in the cubic unit of the conductor, nor the velocity, with which the two electricities displace themselves [sich verschieben] in the current. We can only compare the intensity of the currents by means of the *effects* which they produce.

One of these effects is, e.g., the *decomposition of water*. Sufficient grounds converge, to make the current intensity proportional to the amount of water, which is decomposed in the same time-interval. Accordingly, *that current intensity will be designated as 1, at which the mass-unit of water is decomposed in the time-unit*, thus, e.g., if seconds and milligrams are taken as the measure of time and mass, that current intensity, at which in one second one milligram of water is decomposed. *This measure of current intensity is called the electrolytic measure.*

The natural question now arises, how this electrolytic measure of current intensity is related to the previously established mechanical measure, thus the question, how many (electrostatically or mechanically measured) positive units of electricity flow through the cross-section in one second, if a milligram of water is decomposed in this interval of time.

Another effect of the current is the *rotational moment* it exerts on a magnetic needle, and which we likewise assume to be proportional to the current intensity, conditions being otherwise equal. If a current intensity is to be measured by means of this kind of effect, then the *conditions* must be established, under which the rotational moment is to be observed. One could designate as 1 that current intensity which under *arbitrarily* established spatial conditions exerts an *arbitrarily* established rotational moment on an arbitrarily chosen magnet. When, then, under *the same* conditions, an *m*-fold large rotational moment is observed, the current intensity prevailing in this case would have to be designated as *m*. Precisely the impracticability of such an arbitrary measure, however, has led to the *absolute* measure, and thus in this case the electromagnetic measure of current intensity is to be joined to the absolute measure for magnetism. This occurs by means of the following specification of *normal conditions* for the observation of the magnetic effects of a current:

The current goes through a circular conductor, which circumscribes the unit of area, and acts on a magnet, which possesses the unit of magnetism, at an arbitrary but large distance = R; the midpoint [center] of the magnet lies in the plane of the conductor, and its magnetic axis is directed toward the center of the circular conductor. – The rotational moment *D*, exerted by the current on the magnet, expressed according to mechanical measure, is, under these conditions, different according to the difference in the current intensity, and also according to the difference in the distance *R*; the product R^3D *depends, however, simply on the current intensity*, and is hence, under these conditions, the *measurable* effect of the current, namely, that effect by means of which the current intensity is to be measured, according to which one therefore obtains as *magnetic measure of current intensity* the intensity of that current, for which $R^3D = 1$. – The electromagnetic laws state, that this measure of current intensity is also the intensity of that current which, if it circumscribes a plane of the size of the unit of area, everywhere exerts at a distance the effects of a magnet located at the center of that plane, which possesses the unit of magnetism and whose magnetic axis is perpendicular to the plane; – or also, that it is the intensity of that current, by which a *tangent boussole with simple rings of radius* = *R* is kept in equilibrium, given a deflection from the magnetic meridian

$$\varphi = \arctan \frac{2\pi}{RT}$$

if *T* denotes the horizontal intensity of the terrestrial magnetism.

Here, too, arises the natural question about the relation of the *mechanical* measure of current intensity to this *magnetic* measure, thus the question, how many times the electrostatic unit of the volume of electricity must go through the cross-section of the circuit during one second, in order to elicit that current intensity, of which the just-specified deflection, φ, is effected by the needle of a tangent boussole.

The same question repeats itself in considering a *third* measure of current intensity, which is derived from the electrodynamic effects of the current, and is therefore called the *electrodynamic measure* of the current intensity.

The three measures drawn from the *effect* of the currents have already been compared with one another. It is known that the magnetic measure is $\sqrt{2}$ larger than the electrodynamic, but $106\frac{2}{3}$ times smaller than the electrolytic, and for that reason, in order to solve the question of how these three measures relate to *mechanical* measure, it is merely necessary to compare the later with one of the others.

This was the goal of the work undertaken, which goal was to be attained through the solution of the following problem:

Given a constant current, by which a tangent boussole with a simple multiplier circle or radius = R^{mm} is kept in equilibrium at a deflection $\varphi = \arctan\frac{2\pi}{RT}$ if T is the intensity of the horizontal terrestrial magnetism affecting the boussole: Determine the amount of electricity, which flows in such a current in one second through the cross-section of the conductor, relates to the amount of electricity on each of two equally charged (infinitesimally) small balls, which repel one another at a distance of 1 millimeter with the unit of force. The unit of force is taken as that force, which imparts 1 millimeter velocity to the mass of 1 milligram in 1 second.

2. Solution of this Problem

If a volume *E* of free electricity is collected at an insulated conductor and allowed (by inserting a column of water) to flow to earth through a multiplier, the magnetic needle will be deflected. The magnitude of the first deflection depends, given the same multiplier and the same needle, solely on the amount of discharged electricity, since the discharge time is so short, compared with the oscillation period of the needle, that the effect must be considered as an impulse.

If a constant current is put through a multiplier for a similarly short time, the needle receives a similar impulse, and in this case as well, the magnitude of the first deflection depends *solely on the amount of electricity* which moves through the cross-section of the multiplier wire during the duration of the current.

Now, if in the same multiplier, *exactly the same* deflection were to occur, the one time, when the known amount of free electricity *E* was discharged, the other time, when one let a *constant current* act briefly, then, as can be proven, the amount of positive electricity, which flows during this short time-interval in the constant current, in the direction of this current, through the cross-section, equals *E*/2.

Accordingly, the problem posed requires the solution of the following two problems:

a) measuring the collected amount E of free electricity with the given electrostatic measure, and observing the deflection of the magnetic needle when the electricity is discharged;

b) determining the small time-interval τ, during which a constant current of intensity = 1 (according to magnetic measure) has to flow through the multiplier of the same galvanometer, in order to impart to the needle the same deflection.

If next we multiply $E/2$ by the number which shows how often τ is contained in the second, then the number $\frac{E}{2\tau}$ expresses the amount of positive electricity, which, in a current whose intensity = 1 according to magnetic measure, passes through the cross-section of the conductor in the direction of the positive current in 1 second.

Problem *a* is treated in the following way:

First, with the help of the sine-electrometer, the conditions are determined with greater precision, in which the charge of a small Leyden jar is divided between the jar itself and an approximately 13-inch ball coated with tin foil, which was suspended, by a good insulator, away from the walls of the room, so that from the amount of electricity flowing on the ball, as soon as it was able to be measured, the amount remaining in the little jar could also be calculated down to a fraction of a percent.

The observation consisted of the following:

The jar was charged, the large ball put in contact with its knob; three seconds later, the charge remaining in the jar was discharged through a multiplier[1] consisting of 5635 windings, by the insertion of two long tubes filled with water, and the first deflection φ of the magnetic needle, which was equipped with a mirror in the manner of the magnetometer, was observed. At the same time, the large ball was now put in contact with the approximately 1-inch fixed ball of a torsion balance[2] constructed on a very large scale. This fixed ball, brought to the torsion balance, shared its received charge with [or: gave half its received charge to] the moveable ball, which made it possible to measure the torsion which was required, to a decreasing extent over time, in order to maintain the two balls at a fully determinate, pre-ascertained distance. – From the torsion coefficients of the wire, found in the manner well known from oscillation experiments, and the precisely determined dimensions, the amount of electricity occurring at each moment in the torsion balance could be measured in the required absolute measure, taking into consideration the non-uniform distribution of electricity in the two balls (which consideration was advisable because of the not insignificant size of the balls compared with the distance between them). The observed decrease in torsion also

[1] The mean diameter of the windings was 266 mm; the almost 2/3-mile-long wire, very well coated with silk, was previously drawn through collodium along its entire length, while the sides of the casing were strongly coated with sealing wax. A powerful copper damper moderated the oscillations.

[2] The frame of the torsion balance, in whose center the balls were located, was in the shape of a parallelepiped 1.16 meters long, 0.81 meters wide, and 1.44 meters high. The long shellac pole [Stange], to which the moveable bass was affixed by means of a shellac side-arm, allowed the observation of the position of the ball under a mirror, and then dipped into a container of oil, by means of which the oscillations were very quickly halted.

yielded the loss of electricity, so that it was possible, by means of this consideration, to state how large these amounts would be, if they could already have been in the torsion balance at the moment at which the large ball was charged by the Leyden jar. From the precisely measured diameter of these balls, the proportion of the distribution of electricity between them could be determined (according to Plana's work), so that, by means of the measurement in the torsion balance, without further ado, it was known what amount of electricity remained in the Leyden jar after charging the large ball, and what amount was discharged 3 seconds later by the multiplier. Only one small correction was still required on account of the loss of available discharge, which occurred during these 3 seconds from leakage into the air and through residue formation.

In the following table are assembled the results of five successive experiments. The column headed E contains the amounts of discharged electricity, the column headed s the corresponding deflections of the magnetic needle in scale units, and the column headed φ the same deflections, but in arcs for radius = 1.

Nº	E	s	φ
1	36060000	73.5	0.0057087
2	41940000	80.0	0.0062136
3	49700000	96.5	0.0074952
4	44350000	91.1	0.0070757
5	49660000	97.8	0.0075962

Problem *b* requires knowing the time-intervals τ, during which a current of that intensity denoted 1 in magnetic current measure, must flow through the same multiplier, in order to elicit the deflections φ observed in the five experiments.

The rotational moment, which is exerted by the just-designated currents on a magnetic needle, which is parallel to the windings of the multiplier, is developed in the *second part of the* Electrodynamische Maassbestimmungen *of W. Weber*. This rotational moment is proportional to the magnetic moment of the needle and the number of windings, but moreover is a function of the dimensions of the multiplier and the distribution of magnetic fluids in the needle, for which it suffices, to determine the distance of the centers of gravity of the two magnetic fluids, which, in lieu of the actual distribution of magnetism, can be thought of as distributed on the surface of the needle. The needle always remaining small compared with the diameter of the multiplier, for this distance a value derived from the size of the needle could be posited with sufficient reliability, so that the designated rotational moment D contains only the magnetic moment of the needle as an unknown. – If this rotational moment acts during a time-interval τ, which is very short compared with the oscillation period of the needle, then the angular velocity imparted to the needle is expressed by

$$\frac{E}{K}\tau,$$

where K signifies the inertial moment. The relationship between this angular velocity and the first deflection φ then leads to an equation between τ and φ,

$$\tau = \varphi A,$$

in which A consists of magnitudes to be truly rigorously measured, thus signifies known constants, namely $A = 0.020915$ for the second as measure of time.

Thus, if it is asked how long a time-interval τ a constant current of magnetic current intensity = 1 has to flow through the multiplier, in order to elicit the above-cited five observed deflections, one need only insert their values for τ into this equation. In this way the values in seconds result as

Nº	τ
1	0.0001194
2	0.0001300
3	0.0001568
4	0.0001480
5	0.0001589

If we now divide $E/2$ in the five experiments by the pertinent τ, we obtain

Nº	$\frac{E}{2\tau}$
1	151000×10^6
2	161300×10^6
3	158500×10^6
4	149800×10^6
5	156250×10^6

thus as a mean,

$$\frac{E}{2\tau} = 155370 \times 10^6.$$

The *mechanical measure of the current intensity* is thus proportional

to *magnetic* as $1{:}155370 \times 10^6$,
to *electrodynamic* as $1{:}109860 \times 10^6$
$(= 1{:}155370 \times 10^6 \times \sqrt{1/2})$,
to *electrolytic* as $1{:}\ 16573 \times 10^9$
$(= 1{:}155370 \times 10^6 \times 106\frac{2}{3})$.

3. Applications

Among the applications, which can be made by reducing the ordinary measure for current intensity to mechanical measure, the most important is the determination of the constants which appear in the fundamental electrical law, encompassing electrostatics, electrodynamics, and induction. According to this fundamental law, the effect of the amount of electricity e on the amount e' at distance r with relative velocity dr/dt and relative acceleration ddr/dt^2 equals

$$\frac{ee'}{rr}\left[1-\frac{1}{cc}\left(\frac{dr^2}{dt^2}-2r\frac{ddr}{dt^2}\right)\right],$$

and the constant c represents that relative velocity, which the electrical masses e and e' have and must retain, if they are not to act on each other any longer at all.

In the preceding section, the proportional relation of the magnetic measure to the mechanical measure was found to be

$$= 155370 \times 10^6 : 1;$$

in the second treatise on *electrical determination of measure*, the same proportion was found

$$= c\sqrt{2} : 4;$$

the equalization of these proportions results in

$$c = 439450 \times 10^6$$

units of length, namely, millimeters, thus a velocity of 59,320 miles per second.

The insertion of the values of c into the foregoing fundamental electrical law makes it possible to grasp, why the electrodynamic effect of electrical masses, namely

$$\frac{ee'}{rr}\frac{1}{cc}\left(\frac{dr^2}{dt^2}-2r\frac{ddr}{dt^2}\right)$$

compared with the electrostatic

$$\frac{ee'}{rr}$$

always seems infinitesimally small, so that in general the former only remains significant, when, as in galvanic currents, the electrostatic forces completely cancel each other in virtue of the neutralization of the positive and negative electricity.

Of the remaining applications, only the application to electrolysis will be briefly described here:

It was stated above, that in a current, which decomposes 1 milligram of water in 1 second,

$$106\frac{2}{3}\times 155370\times 10^6$$

positive units of electricity go in the direction of the positive current in that second through the cross-section of the current, and the same amount of negative electricity in the opposite direction.

The fact that in electrolysis, ponderable masses are moved, that this motion is elicited by electrical forces, which only react on electricity, not directly on the water, leads to the conception, that in the atom of water, the *hydrogen atom* possesses free positive electricity, the *oxygen atom* free negative electricity. Many reasons converge, why we do not want to think of an electrical motion in water without electrolysis, and why we assume that water is not in a state of allow electricity to flow through it in the manner of a conductor. Therefore, if we see in the one electrode just as much positive electricity coming from the water, as is delivered to the other electrode during the same time-interval by the current, then this positive electricity which manifests itself is that which belonged to the separated hydrogen particles.

If we take this standpoint, so that we thus link the entire electrical motion in electrolytes to the motion of the ponderable atoms, then it additionally emerges from the numbers obtained above, that the hydrogen atoms in 1 millimeter of water possess

$$106\frac{2}{3}\times 155370\times 10^{6}$$

units of free positive electricity, the oxygen atoms an equal amount of negative electricity.

From this it follows, secondly, that these amounts of electricity together signify the *minimum of neutral electricity*, which is contained in a milligram of water. Namely, if the atoms of water were still to possess neutral electricity beyond their free electricity, then the mass of neutral electricity in a milligram of water would be still greater.

Under the foregoing assumptions, we are also in a position to state the force with which the totality of the hydrogen particles of a mass of water is acted upon in the one direction, the totality of the oxygen particles in the opposite direction.

Imagine, for example, a cylindrical tube of 10/9 square millimeter cross-section, which is to serve as a decomposition cell, filled with a mixture of water and sulphuric acid of specific gravity 1.25, which thus contains in each 1-millimeter segment a milligram of water. Through Horsford, we know the proportional relation of the specific resistance of this mixture to that of silver, and through Lenz, the proportional relation of the resistance of silver to that of copper. In the treatises of the Koenigliche Gesellschaft der Wissenschaften in Goettingen (vol. 5, "Ueber die Anwendung der magnetischen Induction auf Messung der Inclination mit dem Magnetometer"), the resistance of copper is determined according to the absolute measure of the magnetic system. This makes it possible to additionally state, in *absolute magnetic measure*, the resistance which the water (under the influence of the admixed sulphuric acid) exerts in a 1-mm long segment of that cylindrical decomposition cell. This resistance, multiplied by the current intensity, the latter being expressed in magnetic measure, yields the electromotive force in relation to this small cell, likewise in the *magnetic* system of measure. However, the *magnetic* measure of the electromotive force is as many times smaller than the *mechanical*, as the magnetic measure of the current intensity is greater than the mechanical, and since this latter proportion is now known, that electromotive force calculated in magnetic measure can be transformed into mechanical measure simply by division by 155370×10^{6}. The number which results then signifies the *difference between the two forces*, of which in the direction of the current, the one acts to move *each single unit* of the free positive electricity in the hydrogen particles, the other to move *each single unit* of the free negative electricity in the oxygen particles, and therefore, in order to obtain the *entire force at work*, this number must still be multiplied by the total of units of the free positive or negative electricity, which is contained in the 1 millimeter-long wet cell, that is, in 1 milligram of water, namely, by

$$106\frac{2}{3}\times 155370\times 10^{6}.$$

If one carries out the calculation and presupposes that current intensity, at which 1 milligram of water is decomposed in 1 second, then one obtains a force difference

$$= 2\times\left(106\frac{2}{3}\right)^{2}\times 127476\times 10^{6},$$

in which the unit of force is that force, which imparts to the unit of mass of 1 milligram a velocity of 1 millimeter in 1 second. Thus, if one divides by the intensity of gravity = 9,811, one obtains this force difference, expressed in weight

$$= 2\times 147830\times 10^{6} \text{ milligrams} = 2\times 147830 \text{ kg} = 2\times 2956 \ \textit{Centner}$$

under the influence of gravity.

This result can be expressed in the following way: *If all hydrogen particles in 1 milligram of water were linked in a 1 millimeter-long string, and all oxygen particles in another string, then both strings would have to be stretched in opposite directions with the weight of 2,956 hundredweight, in order to produce a decomposition of the water at a rate such that 1 milligram of water would be decomposed in 1 second*.

One easily convinces oneself, that this stretching remains the same for a cell of 1 mm length but a different cross-section, but that it must be proportional to the length of the cell, and also proportional to the current intensity, that is, to the velocity of the electrolytic separation.

If, in the wet cell described above, we now see a pressure on the totality of hydrogen particles of the weight of 2,956 *centner*, and if no acceleration of motion occurs, which motion must, however, amount to 1,759 million miles per second, but rather the hydrogen continues with the constant velocity of ½ millimeter per second, then we are compelled to assume, that a force would be acting counter to the decomposition of the water, a force which increases with the velocity of the decomposition, so that in general, only that velocity of decomposition remains, at which the *force of resistance* is equal to the electromotive force, so that its effect on the totality of hydrogen particles in the milligram of water in the foregoing case likewise would equal the weight of 2,956 hundredweight. Namely, in that case, the ponderable particles would uniformly flow forth with the velocity attained.

It is natural, to seek the basis for this force of resistance in the *chemical forces of affinity*. Even though the concept of *chemical affinity* remains too indeterminate, for us to be able to derive from it, how the forces proceeding from this affinity increase with the *velocity of the separation*, nevertheless, it is interesting to see what colossal [*ungeheuren*] forces enter into operation, as are easily elicited by electrolysis.