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The Undulatory versus the Corpuscular Theory of Light: The case of the Doppler Effect

(how to win without playing the game)

At the beginning of our century the undulatory theory of light was so firmly shared that the proposal by Einstein of the light quantum (1905) provoked-as is well known-a hostile and tenacious reaction. From the theoretical point of view, this reaction was founded on the capacity of the undulatory theory to explain the macroscopic behaviour of light, diffraction and interference phenomena included. Einstein himself was very cautious in the building of the concept of the light quantum, initially endowed with only an energy content $h\nu$, then-twelve years later (1917)-with a linear momentum $h\nu / c$.

The so-called Doppler effect arose in the context of undulatory, acoustical and optical descriptions. The electromagnetic theory of light reinforced the belief that the optical Doppler effect was an undulatory phenomenon. Of course, this statement is ambiguous. In fact, it may signify:

a) the optical Doppler effect *must* be described within an undulatory theory;
b) light *is* constituted by waves (ontological statement) and, consequently, the Doppler effect *is* an undulatory phenomenon;
c) both a) and b).

The situation was largely represented by case c). In fact only a shared image of the physical world, of which electromagnetic waves are fundamental *existing* components, may explain the fate of the corpuscular theory of light as applied to the Doppler effect.

In 1922 Schrödinger wrote a paper on "The Doppler principle and Bohr's frequency condition".[1] For the hypothesis of light quanta it was a period more intriguing and vital than usually acknowledged: creativity, mistakes, hesitancies, missed links and overlooked breakthroughs were amazingly mixed up. Before Schrödinger, Emden used for the first time the linear momentum of light quanta for a corpuscular non-relativistic treatment of the Doppler effect.[2] A year before,

[1] SCHRÖDINGER (1922).
[2] EMDEN (1921).

Försterling mixed up light quanta and relativity and dealt also with the Doppler effect. His application of special relativity was wrong; but, in any case, he would not have obtained anything durable, because his light quanta were devoid of linear momentum.[3] De Broglie came to the threshold of Bose's derivation of Planck's formula for black-body radiation and missed it for the same reasons-an (infinitesimally) small mass attributed to the light quantum-that would have led him to the formulation of the idea of the *fictitious* wave associated to a particle.[4]

Schrödinger's paper represents an outstanding breakthrough: it contained the first *corpuscular* and *complete* description of the Doppler shift of spectral lines. Schrödinger's relativistic treatment of the system *atom+light quantum* preceded that of Compton[5] and Debey[6] applied to the system *photon+electron*. He wrote down the relativistic equations for the conservation of energy and linear momentum for the system *atom+light quantum*. After some calculations he got the equation:

$$\nu^* = \nu \sqrt{\frac{(c - \mathrm{v}_1 \cos\theta_1)(c - \mathrm{v}_2 \cos\theta_2)}{\sqrt{(c^2 - \mathrm{v}_1{}^2)(c^2 - \mathrm{v}_2{}^2)}}} \qquad (1)$$

with:

$$\nu^* = \frac{E_1{}^2 - E_2{}^2}{2h\sqrt{E_1 E_2}} \qquad (2)$$

where: E_1 and E_2 are the rest energy of the atom before and after the emission; ν is the frequency measured by the spectrograph; v_1 and v_2 are the velocities of the emitting atom with respect to the spectrograph before and after the emission; θ_1 and θ_2 the angle formed by the atom velocity with the direction of flight of the light quantum before and after the emission. All these quantities are evaluated in the reference frame of the spectrograph. Strangely enough, Schrödinger stopped at this equation. Few further passages lead, in fact, to the equation:

[3] FÖRSTERLING (1921).
[4] DE BROGLIE (1922).
[5] COMPTON (1923).
[6] DEBYE (1923).

$$\nu = \nu_0 \frac{\sqrt{1 - \frac{v_1^2}{c^2}}}{1 - \frac{v_1}{c}\cos\theta_1} \qquad (3)$$

where:

$$\nu_0 = \frac{\Delta E}{h}\left(1 - \frac{\Delta E}{2E_1}\right) \qquad (4)$$

where $\Delta E = E_1 - E_2$ is the energy difference between the two quantum levels involved in the electronic transition. Equation (3) is formally identical to the standard equation for the Doppler effect of the undulatory theory. However, the meaning of ν_0 is radically different in the two theories. In the undulatory description ν_0 is the frequency measured by a spectrograph that sees the atom at rest. The theory is not able to say if at rest before or after the emission for the simple reason that such a distinction is meaningless, in conformity with the idea that the atom emits a spherical wave and, therefore, it cannot acquire linear momentum during the emission. In the corpuscular description instead, ν_0 has the following meaning: it is the frequency measured by a spectrograph that sees the atom at rest *before* the emission; furthermore, the corpuscular description introduces the term $\frac{\Delta E}{2E_1}$ that represents the recoil energy taken up by the atom during the emission.

It can be shown that Schrödinger's treatment can be easily applied also to the case of absorption. The final equation is again equation (3) where now ν_0 is the frequency that the atom absorbs if it is at rest, in the reference frame of the spectrograph, before the absorption and, obviously:

$$\nu_0 = \frac{\Delta E}{h}\left(1 + \frac{\Delta E}{2E_1}\right) \qquad (5)$$

where, now, $\Delta E = E_2 - E_1$.

Why did Schrödinger stop at equation (1)? First of all, it must be emphasised that the physics of the phenomenon and the ensuing calculation clearly show that the velocity of the atom after the emission is determined once its velocity before the emission is known. The answer might be that Schrödinger was pleased by the symmetry of equation (1) with respect the two velocities and did not go further. His final comment seems to give some support to this suggestion:

> *This* [equation (1)] *differs from the usual relativistic Doppler formula only because the multiplying factor of* v *is the geometric average between the two values that should appear in that formula for the initial and final velocity*.[7]

Though equations (4) and (5) are currently used today by physicists, the original derivation by Schrödinger has been completely forgotten: we do not know of any research paper that quotes Schrödinger's. The corpuscular treatment of the Doppler effect has been only a little more fortunate: similar derivations (without quotation) can be found, for instance, in the books by Møller[8] and French.[9] However, the corpuscular approach to the Doppler effect has not been applied in research papers also when it was strictly necessary or its use advisable.

As an illustration, let us refer to the works on time dilation experiments carried out with atoms in flight or with the Mössbauer effect. We have made a rather extensive study of these and related issues in a paper to be submitted for publication.[10] We can summarise the question in the following way:

1. It is assumed that atoms (or nuclei) emit periodic waves. Our acquired knowledge (theoretical and experimental) tells us instead that atoms emit quanta and that this emission is directional. Our acquired knowledge tells us also that the undulatory theory of light works only for large intensities and that it can be used only when individual emission or absorption processes sum up to give a macroscopic datum in the detector and, consequently, can be neglected in the theoretical description.
2. When the physical state of the emitting or absorbing atom (or nucleus) is changed for some reason (for instance because it is in a gravitational field or it is accelerated), the use of the corpuscular description is mandatory, because it is the only one that can describe how to take into account these physical changes.

To briefly illustrate these points, let us first consider the experiments with emitting atoms flying in inertial motion.[11,12,13] The physical state of the emitting atoms is not influenced by their inertial motion. Moreover, the shift of the spectral line is observed by looking at the macroscopic images of the entrance slit of the spectrograph on a photographic plate: the atoms composing this plate can be changed without changing the essential feature of what is observed (the position of the lines on the plate). In this case, the two conditions for the application of the undulatory description are satisfied. However, the use of the undulatory description implies the assumption that atoms emit periodic waves and can be considered as clocks. These assumptions find their roots in nineteenth century physics. In

[7] SCHRÖDINGER (1922), p. 303.
[8] MØLLER (1972), pp. 401-7.
[9] FRENCH (1968), pp. 197-9.
[10] BONIZZONI and GIULIANI to be submitted.
[11] IVES and STILWELL (1938).
[12] OTTING (1939).
[13] MANDELBERG and WITTEN (1962).

particular, the emission of periodic waves by matter was explained in terms of periodic motions of charged particles having the same frequency of the waves. However, firstly Bohr's quantum condition has broken the link between internal motions and emitted frequencies; then quantum mechanics has rule out any possible link among them. It seems unwise to use disputable and *superfluous* hypothesis for testing time dilation predictions by special relativity. The experiments with flying atoms are correctly described by Schrödinger's treatment and the experiments should thus be considered as a corroboration of relativistic dynamics.

As a second example, let us consider the experiments based on the Mössbauer effect and performed with the source and absorber on the same diameter of a rotor at different distances R_s, R_a from the centre. The experimental results are asymmetrical in the exchange between the position of the source and absorber. They are described by the equation (for small velocities):

$$\frac{\varepsilon_a - \varepsilon_s}{\varepsilon_s} \approx \frac{\Omega^2}{2c^2}\left(R_s^2 - R_a^2\right) \tag{6}$$

where ε_a and ε_s are the energies of the absorbed and emitted quantum, Ω is the rotor angular velocity and c the speed of light. Equation (6) is derived within a general relativity approach: notice its asymmetry depending on the sign of $\left(R_s^2 - R_a^2\right)$.

The interpretation that can be found in the papers reporting the experiments[14,15,16,17] claims that equation (6) can also be derived within a special relativity approach. It can be shown that this is not true, as it can be presumed on the basis that in special relativity two inertial observers are perfectly equivalent. As a matter of fact, a sound special relativity treatment yields, instead of equation (6), the equation:

$$\frac{\varepsilon_a - \varepsilon_s}{\varepsilon_s} \approx -\frac{\Omega^2}{2c^2}\left(R_s - R_a\right)^2$$

which does not depend on the sign of $\left(R_s - R_a\right)$.

From the physical point of view, the general relativity approach implies a change in the energy of the quantum emitted or absorbed, due to the pseudo–gravitational potential arising from acceleration. In the papers quoted above, the special relativity description is based, instead, on the assumption that nuclei can be considered as clocks since they are supposed to emit periodic waves.

[14] HAY et al. (1960).
[15] CHAMPENEY et al. (1961).
[16] KÜNDIG (1963).
[17] CHAMPENEY et al. (1965).

Though sketchy, the above analysis shows that the undulatory theory of light has played-in the case of the Doppler effect-a role that has largely gone beyond its explanatory capacities. On the other hand, the corpuscular theory has been substantially neglected. This is an impressive case of the role that a shared image of the physical world can play in the theoretical description of phenomena.

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