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Electrodynamics of Enrico Fermi between Research and Didactics

1.“Elettrodinamica” of Enrico Fermi

Enrico Fermi’s “Elettrodinamica” is an unpublished essay written in Italian for didactic purposes in 1925. It is made up of four chapters: the first one deals with *Electrostatics*, the second with *Magnetostatics*, the third with *Electromagnetism* and the *problem of the electromagnetic mass*, while the last section concerns the *Theory of Relativity*.

Three different versions of “Elettrodinamica” are at present known:
– the *first*, at the University of Pisa, is a handwritten adaptation of Fermi’s original essay. It cannot be attributed to Fermi himself.
– the *second*, in the Library of the Department of Physics of the University “La Sapienza” in Rome, is a handwritten copy made by Adelino Morelli.
– the *third* version, in the Chicago University Library, is Enrico Fermi’s original copy. It is a typed version that was left in Chicago, after Fermi’s death in 1954, by his wife Laura Fermi. This version contains all the arguments discussed in the program of the course of mathematical physics that Fermi taught at the University of Florence in 1925:

i) Vectorial field theory: Gradient - Gauss’ theorem - Flux - Divergence - Stokes’ integral theorem - Rotation.
ii) Elements of Potential Theory: Coulomb’s law - Flux of E - Poisson’s equation - Harmonic functions - Green’s formula - Integration of Poisson’s equation - Shell potential - Double shell - Dipoles - Electrostatic energy - Capacitors.
iii) Electrostatics of dielectrics: Polarization of molecules - Field of polarized dielectrics - Force and induction - Dielectric costant - Electric energy in dielectrics - Molecular theory of dielectrics.
iv) Magnetostatics: magnets - magnetic quantities.
v) Electromagnetism: Currents and magnetic fields - Laplace’s law - Equivalence between magnetic foils and currents - Forces and currents.
vi) Induction: Electromagnetic induction - Self-induction - Summary on the electromagnetic laws - Maxwell’s equations - Plane waves - D’Alembert’s equation - Velocity of light - Elements of the waves - Flux of energy - Radiation pressure -

Reflexion - Refraction - Kirchhoff's theorem - Scalar and vectorial potential - Retarding potentials - Irradiation of an oscillator - Irradiation in general.
vii) Passage of Light through Matter: Absorption of light in a wire - Dispersion - Diffusion.
viii) The electromagnetic masses: Charge field in uniform motion - Magnetic mass.
ix) Special Theory of Relativity: Frames of reference - General postulates of the theory of relativity - Lorentz-Einstein's Transformation - Consequence of transformation - Addition of velocities - Drag of light - Lorentz's Trasformation as a rigid rotation - Geometry of four-dimensions spaces - Vectorial calculus in four-dimensions spaces - The transformation of electric quantities - Four-potential - Kinetic energy.

2. Description and Analysis of "Elettrodinamica"

As the list of the arguments attests, the essay "Elettrodinamica"[1] contains only classical electrodynamics and relativity. There is nothing about quantum mechanics and quantum electrodynamics.

One of the most evident characteristics of "Elettrodinamica" is the importance accorded to the mathematical description of physical problems. The mathematical approach is evident from the outset: in the first part of "Elettrodinamica" there is a long and detailed discussion of differential vectorial operators.

The same arguments and the same mathematical description can be found in the first chapter of H.A. Lorentz's *The theory of electrons*.[2] It is important to observe that, even if "Elettrodinamica" was conceived as part of a course of mathematical physics, Fermi uses mathematics in a highly sophisticated way to treat the different arguments pragmatically, obtaining the most significant scientific results with an analysis of the essential aspects of physical phenomena.

In the first part, *Electrostatics*, the most important argument Fermi discusses is the physics of dielectrics and, in particular, the electrostatics of dielectrics.

He sets out from Maxwell's dielectric displacement and, using the theory of electric potential, describes the free electricity potential and the charge potential in a capacitor.

The same approach to dielectrics is to be found in Pauli's "Elettrodynamics"[3] in the chapter entitled "Phenomenological analysis of dielectrics". Both Fermi and Pauli conclude the study of electrostatics of dielectrics with the Clausius-Mossotti equation.

In second part, *Magnetostatics* is described by analogy with electrostatic phenomena: the magnetic field is introduced, the fundamental quantities of magnetostatics are defined and the magnetostatic laws obtained by analogy with electrostatics are described.

[1] In this work only the version of "Elettrodinamica" preserved at the Chicago University Library is dealt with. For a detailed description of "Elettrodinamica", see also: Enrico Fermi, "Elettrodinamica", Walter Joffrain ed., Hoepli, 2002.
[2] LORENTZ (1952).
[3] PAULI (1949).

In the third part of "Elettrodinamica" there is an interesting analysis of *Electromagnetism*; Fermi's statement and arguments are the same as those found in the chapter "Teoria elettromagnetica della luce" in the volume *Introduzione alla fisica atomica.*[4]

The difference is that in "Elettrodinamica" Fermi can describe the electromagnetic phenomena directly because he has already introduced the main elements of electrology in the chapter on magnetostatics. Fermi opens the discussion with the equation of continuity of electricity and goes on to introduce the law of electromagnetic induction and self-induction and, in conclusion, he proposes an exercise, the only exercise in "Elettrodinamica":

> *calculate and discuss the oscillations of an oscillator, that is a circuit with resistance, capacity and self-induction.*[5]

A long chapter follows containing an analysis of Maxwell's equations and the propagation of electromagnetic waves with the description of phenomena like reflexion, refraction, diffusion and absorption of radiation.

At the end of this chapter, Fermi calculates the electromagnetic momentum of an electromagnetic wave and discusses the problem of electromagnetic mass.

3. Fermi and the Problem of Electromagnetic Mass: Between Research and Didactics

One of the most interesting aspects of "Elettrodinamica" is the way it handles the problem of electromagnetic mass at the end of the chapter of Electromagnetism.
Fermi describes and calculates the value of the electromagnetic mass of a system of electric charges without considering an important aspect: the difference between the electromagnetic mass obtained with the pre-relativistic theory and the value proposed by the theory of relativity.

Fermi was well aware of this question as is clear from the articles[6] he published before "Elettrodinamica", which resolve the problem of the difference between the pre-relativistic and the relativistic result.

We conclude that Fermi, though he knew and had already solved the problem of the correction to make to the electromagnetic mass term, preferred to offer his students the pre-relativistic result as well.

We may suppose that this choice derives from the didactic aim of the essay. Fermi clearly prefers not to describe the relativistic correction to the mass term because he had not introduced in "Elettrodinamica" the physical and mathematical

[4] FERMI (1928).
[5] The original form is: "Come esercizio si calcolino e si discutano le oscillazioni di un oscillatore, cioè di un circuito contenente una resistenza, una autoinduzione e una capacità" (Fermi, *Elettrodinamica* (Chicago version), p. 50).
[6] FERMI (1921).

notions necessary for an understanding of the problem: the contradiction between the electrodynamic and the relativistic theory of the electromagnetic mass.

In addition to the methodological explanation of this choice there may be another justification. It is connected with Fermi's prudent approach when he discusses the delicate subject of electromagnetic mass, concerning which the physicist community in the twenties had expressed different opinions. Only two years had passed between the solution Fermi gave to the problem of electromagnetic mass in his article[7] and the draft of "Elettrodinamica". Fermi's ethical correctness led him to face in didactics only arguments already accepted.

The consequence of this separation between research and didactics is that Fermi is unable to propose the new relativistic results to his students. Neither can he illustrate the importance of the interpretation of mechanics through electromagnetism as a consequence of the electromagnetic explanation of inertia.

Pauli's "Elettrodynamics"[8] also contains no reference to a relativistic correction to electromagnetic mass. Fermi's and Pauli's choice is not that of Feynman in *The Feynman Lectures on Physics.*[9] Despite the fact that Feynman chooses a didactic approach to the problem of electromagnetic mass, he introduces the relativistic corrections and analyses their implications.

4. The Theory of Relativity in "Elettrodinamica"

The presence of a chapter dedicated to the theory of relativity makes "Elettrodinamica" of considerable scientific interest and underlines the importance Fermi attaches to this topic even in an essay conceived for didactic purposes.

This fact reveals Fermi's openess towards the teaching of new sciences and confirms his propensity towards didactics and education.

The presence of a chapter on the theory of relativity in "Elettrodinamica" is even more significant if we consider that in Italian universities in the twenties this subject was seldom touched upon. This precocious interest in relativity also reveals the breadth of knowledge of the young Fermi, who was essentially self taught, given the backward situation of theoretical physics in Italy in the twenties.

Fermi's work should however be included in the tradition of Italian physics of the beginning of the century, despite the fact that it does not represent a continuation of that tradition. It did indeed represent a significant turning-point both for his great scientific contributions and for the research program.

An analysis of the chapter "The theory of relativity" in "Elettrodinamica", shows how Fermi prefers a geometrical description of relativity, in accordance with the Italian mathematical tradition in physics of the twenties. Fermi's didactic ability can

[7] Ivi, pp. 201-5.
[8] PAULI (1949).
[9] FEYNMAN, LEIGHTON and SANDS (1963).

be appreciated in the way he insists on less intuitive geometrical concepts like space-time in four-dimensions and the geometry of four-vectors.

5. Conclusion

An interesting characteristic of "Elettrodinamica" is the great importance accorded to the mathematical description of the physical problems discussed.

Fermi uses mathematics to treat the different arguments pragmatically, obtaining the most interesting scientific results by analysing the essential aspects of physical phenomena.

One of the most intriguing aspects of "Elettrodinamica" is its treatment of the problem of electromagnetic mass. Fermi deals with it without taking into consideration the difference between the result obtained with the pre-relativistic theory and the new value proposed by the theory of relativity. Fermi was aware of and had already solved the problem of the contradiction between the electrodynamic and the relativistic theory of electromagnetic mass, but he preferred to offer to his students the pre-relativistic result as well.

This choice is the result of the didactic aim Fermi decided to pursue when he wrote the essay, and of his ethical correctness towards didactics.

The chapter dedicated to the theory of relativity makes "Elettrodinamica" an interesting scientific document to underline the importance Fermi accords to this argument in an essay realized for didactic purposes. This confirms Fermi's open-minded attitude towards the teaching of new sciences and reveals his propensity towards didactics and education.

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