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From Electrodynamics to Quantum Electrodynamics: a History of the Symbolic Representation of Physics Laws

1. Introduction

In the two hundred years from the mid nineteenth century to the mid twentieth century, physics underwent a remarkable revolution in its mathematics. Let us consider that as a written language, mathematics (herewith shortened: math) was represented by symbols; it can be argued that the symbolisation of physics laws, i.e., the form of the written math expressions, usually labelled as physical formulas, was not neutral to the conceptual content of laws. In this paper I wish to examine how, in the course of the history of physics, symbols interweaved with their conceptual counterpart, contributing to the conceptual evolution of physics. I am aware that my study represents a first approach to what I consider an important and scarcely explored topic in the philosophical history of physics, i.e., the relationship between concepts and their graphical representation.

Historical scientific literature has been recently interested in a more general problem concerning the foundational assumptions of the math chosen to represent the physical world.[1] In this paper I intend to stick to a more circumscribed topic, by inquiring how electrodynamics and electromagnetism were the two research areas mainly responsible for the transformation of symbols, a special aspect of the mathematisation of physics which, in the thirties of the nineteenth century, contributed to the construction of quantum electrodynamics.

Starting at least from Gauss's and Weber's contributions, physicists debated the problem of the meaning of symbols in their equations, passing from the idea that they represented merely rational numbers to the different view that the symbolisation concerned physical entities in themselves.

One can see that in Gauss's and Weber's electrodynamics symbols represented mere rational numbers, i.e., measures of physical quantities, such as length, charge, etc. It was Wilhelm Weber's merit in the 1850's to give a more rational order to this

[1] ISRAEL (1996); DRAGO (1990).

special form of symbolisation by introducing systems of units. He thus succeeded in expressing electrodynamic laws in the form of algebraic equations, in place of mere proportionality relations, the formerly accepted representation of physics laws. In a short time, Maxwell made a further advance by representing symbols in the form of dimensional quantities. But this innovation was soon extended, introducing more complex forms of representations such as *gradients*, *divergency*, *rotors* and the like, generally labelled as vectorial operators.

It was therefore conceived that two different kinds of meanings and theoretical roles were to be attributed to symbols, one related to the measurements of physical quantities, the other to the physical quantities per se (e.g., of being *scalars*, or *vectors*, *tensors*, etc.). Notice that, due to this distinction of roles, new types of numbers, beyond the initial rational numbers, were introduced in physics, such as imaginary numbers, *quaternions*, *operators* and *matrices*.

Thus, starting from the middle of the nineteenth century, the meaning of symbols in the mathematical representation of physics laws passed from the description of metrological properties of physics objects to the representation of higher levels math-physical properties. This process continued and was enhanced in modern theoretical physics by the introduction of *spinors*, *matrices*, *creation* and *destruction operators*, Dirac's *q* and *c* numbers, and so on.

In the thirties of the twentieth century, Adrian Dirac was one of the great physicists who completed the transformation of symbols, being aware of the profound modification in methods and scope of the math-physics relationship therewith entailed.

2. From Rational Numbers to Vectorial and Tensorial Calculus: The Classical Evolution of the Symbolic Representation of Physics

No doubt that, through his work in earth magnetism, Carl Friedrich Gauss (1777-1855) started a revolution in electrodynamics and in physics. Throughout 1832 Gauss worked to develop and test a method for measuring the quantity and direction of the earth's magnetic intensity, independently of the characteristics of the measuring compass.[2] In fact, the methods used until then were largely unreliable mainly because the measures were dependent on the particular magnetic moment of the compass employed and were variable in time due to variations in this moment. Due to both features (independence of instrument and independence of location), Gauss called the units "absolute". Wilhelm Eduard Weber (1804-1891) was, for a major part of his life, a collaborator and a friend of Gauss at the University of Göttingen. The path to Weber's electrical researches lay through Gauss's magnetism at Göttingen. In fact, Gauss's and Weber's magnetic interests soon extended to the exploitation of the magnetic techniques in the new field opened by Faraday's recently discovered electromagnetic induction. In 1843, Weber had become particularly concerned with Ampère's electrodynamics. When he returned

[2] GAUSS (1832).

to Göttingen in 1849 he had already contributed to electrodynamics important results which were published in 1846.[3]

Weber's research culminated in his discovery of a fundamental law of electrodynamic action, which he presented in his influential 1846 paper "Electrodynamic Measures on a General Fundamental Law of the Electric Action". The law was fundamental in the sense that the electric action applied to electric "masses" themselves rather than to their ponderable carriers, the conducting wires.

A definition of electrodynamic and electromagnetic units of current intensity, independently from Gauss's method of current-magnet interaction, was included among the results of Weber's works, thus providing a simpler basis for the absolute measure of electric current in terms of fundamental mechanical units.[4]

In order to attribute the physical meaning of a velocity to a proportionality constant c, figuring in his equation, Weber resorted to considerations on the meaning of symbols in his fundamental law. They represented rational numbers, i.e., measures of physical quantities. Weber accurately measured the value of c, and found the ratio between the force exerted by a given quantity of frictional electricity standing in a condenser and the force it exerted when flowing in a wire. He thus solved the long known problem of the way of linking electrostatics with electrodynamics, a problem that Ampère had bypassed and that intrigued Faraday in his researches. He remarked that in his fundamental law, c^2 represented the ratio between the electrostatic (charge-charge force) and the electrodynamic (current-current force) force in mechanical units. Thus his theory explained why the electrodynamic interaction in electrostatic units between current carrying wires appears to be infinitesimally small in comparison with the electrostatic interaction ee'/r^2; so that, in most cases, the former remains insignificant with respect to the latter in a galvanic current, where the electrostatic force is completely eliminated on account of the neutralisation of positive and negative carriers of electricity.[5]

Although the German scientist did not identify c with the velocity of light, as Maxwell did in 1862, Weber added c in the role of a second universal constant to Newton's gravitational constant. Besides deeming c important for the reasons above, Weber attributed to it a metrological import as an indication of a new connection between the space and time units, in a new formulation of the mechanical units of space and time, which reduced the fundamental mechanical quantities from three to two (i.e., space and mass).

From the viewpoint of this paper, it is important to underline that Gauss' and Weber's achievements mainly derived from their metrological approach. This approach had also general consequences for the methods of physics. In fact, due to the above systematic organisation of units into absolute systems, electrodynamic laws could be written in the form of analytical equations, including physically significant proportionality constants. Historically, this representation was extended

[3] WEBER (1893).
[4] For a general view of Weber's metrological contribution: D'AGOSTINO (1996).
[5] *Ibid.*, pp. 19-21.

and became a standard for physics: equations replaced the proportionality relations and purely local numbers, the form in which physics laws were symbolised in the writings of Coulomb, Fresnel and others physicists of the beginning of the nineteenth century. This richer representation of laws had the important methodological consequence that theories could be used for guessing the result of experiments on the measure of quantities. Let us remark that, a few decades afterward, Heinrich Hertz considered this predictive power of theory as the main feature of theoretical physics.[6]

I argue that it is not marginal that J.C. Maxwell, along with Kelvin and Hamilton, introduced new symbols in the mathematical codification of physical law. As a consequence of Maxwell's choice of two "mathematical systems of units", symbols in his equations did not represent pure numbers but had dimensions obeying a homogeneity principle. Thus, dimensions had an important role in Maxwell's master work, *A Treatise on Electricity and Magnetism*, for his electromagnetic theory of optics was supported, among the others, by a proof directly connected with Maxwell's dimensional approach[7] to the representation of quantities.[8]

In his important paper, "On the mathematical classification of physical quantities",[9] Maxwell announced a new way of relating math to physics: his classification was founded on the mathematical or formal analogies between physically different quantities. It differed from the traditional classification based on quantities which differed for "the matter to which they belong". Maxwell's new classification was advantageous for establishing analogies between quantities belonging to different phenomena, such as quantities in gravitation and in the steady conduction of heat, so that "one theory can be transferred to solve problems in the other" (p. 258).[9] Quantities were classified in accord with their vectorial and tensorial representations, i.e., for their intrinsic mathematical features, which, in as much as mathematical, were especially adapted to generalisation. Their generalisation afforded in turn new physical interpretations.

Maxwell's classification implied a new meaning attributed to physical symbols, each symbol consisting of two factors, a numerical quantity times a standard quantity of the same kind with that to be defined (p. 258).[9] In the case of energy, Maxwell remarked that this quantity could be defined in two different ways, either as a squared velocity times the mass, or as the product of a quantity of motion and a velocity, both factors being vectorial quantities. The second definition proved the most successful for the new science, because it afforded a physical interpretation: one factor "is conceived as a tendency towards a certain change, and the other as the change itself" (p. 260).[9] In order to conform to the aforementioned mathematical

[6] HERTZ (1956), Introduction, pp. 1-2.
[7] The heuristc value of this approach remains untouched in spite of the fact that Maxwell's ideas on this point have been criticized and the method rejected by Helmholtz, Hertz, Sommerfeld and others. A hint, in my opinion, of the lack of a neat distinction between a logic of research and a logic of proof in history of physics.
[8] D'AGOSTINO (1996), p. 37 *passim*.
[9] MAXWELL (1874), pp. 257-66.

classification, Maxwell invented and used new symbols and new names (such as the vectorial operators of curl, convergence, concentration, etc.[10]

As an example of the advantages of the new classification, let us take Maxwell's definition of forces as vectors referred to unit of length, and of fluxes as vectors referred to unity of area: in the theory of ordinary fluids, the definition of flux velocity, either as a forces or as a flux, was conceptually indifferent, but in developing a more complex theory, as in gas diffusion, only the second definition could account for the phenomenon "where one fluid has a different velocity from another in the same place" (p. 260).[9] Notice that the second definition is symbolically represented by Maxwell through a new vectorial operator, today labelled as "Divergency" (Maxwell: "Convergency").

Let us conclude this chapter with the remark that, in a section of his masterwork *A Treatise on Electricity and Magnetism*, Maxwell used the Quaternions Calculus invented by W.R. Hamilton.[11] Heaviside and Gibbs saw in the mathematics of the Treatise the first example of a vectorial analysis.[12]

3. From Operators to Imaginary Numbers, and Matrixes

In the last two centuries, the introduction of operators marked important innovations in both the conceptual and the technical structures of theoretical physics.

According to Jammer (p. 224)[13] the initial formulation of the concept was presented in 1837 by Robert Murphy, and the use of operators to help solve differential equations was proposed in 1859 by George Boole. He thought of operator calculus (herewith: OC) as a method for checking the correspondence between "operations possible in thought" and symbolic operations. The elaboration of symbolic techniques for the solution of differential equations[14] became an important subject of research in the second half of the last century, especially among British mathematicians such as Charles Graves. In fact, Oliver Heaviside took inspiration from Graves (p. 226),[13] and in 1893 he applied OC to electromagnetic problems and particularly to electric networks. He objected to mathematical rigour as an impediment to a their profitable application, a typical physicist's initial approach to OC, later on imitated by several physicists.[15]

Another example of the conceptual change required to introduce new symbols is offered by Schrödinger's resistance, in the 1920's, to introduce imaginary quantities in

[10] Some are Maxwell's originals, other are Kelvin's and Hamilton's.
[11] MAXWELL (1954), Articles 11, 303, 490, 522, 618.
[12] CROWE (1967), p. 252.
[13] JAMMER (1966).
[14] As, for example, by Hargreave, Gregory, Brouwin, Carmichael, Forsyth (p. 226).[13]
[15] For example, Norbert Wiener confessed that in 1926 he presented to Born his method of generalising matrix calculus in the form of OC, but the latter objected to the soundness of Wiener's limited method which could have met Hilbert's criticism (JAMMER (1966), p. 221).[13]

his wave equation.[16] In fact he initially tried by any means to avoid the appearance, in his equation, of the immaginary number *i*. Only later did Schrödinger recognise the fundamental role of complex quantities in his wave equation and in the second quantisation method The mathematics and physics of OC, rather distinct until 1925-26, converged when Born and Wiener, in Cambridge Mass., generalised quantum matrix mechanics into an OC, thus initiating the new era of cooperation between physics and math. (p. 220 *passim*).[13] A second way of generalising quantum matrix mechanics, initiated by Heisemberg, was completed by Dirac in 1925.[17]

Dirac was convinced of the indispensability of using Hamilton's mechanics for the study of atomic physics, and unsatisfied with Heisenberg's approach, he tried to adapt Heisenberg's formalism to the Hamiltonian's. He succeeded in recasting "Heisenberg's mechanics into an algebraic algorithm on the basis of which he expected to derive all the formulas of the quantum theory without any explicit use of Heisenberg products (Matrices)" (p. 229).[13] Dirac's 1925 paper presented incomplete results but, in a second 1926 paper, he presented a "quantum algebra", including a new type of math entities which he called *q* numbers, in contradistinction to c numbers, the rational numbers of classical physics.[18] Although Dirac's new algoritm could not yet account for non periodic systems – at difference with the Born-Wiener operator calculus – his theory was able to account for the complete frequencies of hydrogen atom, a result that had escaped Born and Wiener. Dirac had thus established "one of the most profound and useful relations between quantum mechanics and the classical Hamilton-Jacobi formulation of mechanics" (p. 229).[13] By promoting the fundamental dynamical variables of a corresponding classical system of microscopic particles, interacting istantaneously, into operators with specific commutators, the OC interpretation of Hamilton's equations had produced rules, for transforming a system, described initially in particles language, into a wave picture (p. VII).[19]

The success of the new mathematical techniques was assimilated by Jammer (p. 293)[13] to the development in classical mechanics of the canonical formalism in the works of Hamilton, Jacobi, Poincarè and Appel, a formalism that transformed Newtonian mechanics from a science of masses and forces to a pure formal structure of fundamental mathematical equations for canonical variables, which included a classical wave theory.

4. Dirac's Symbolic Method and the Confluence of Math with Physics

While in the new quantum mechanics there existed the above mentioned general method for transforming physically the mathematics of a system of particles interacting instantaneously into the complementary wave picture, no adequate

[16] C.N. YANG underlines Schrödinger's reluctance at that time to use complex numbers in the role of physical quantities (YANG (1987), p. 547).
[17] DIRAC (1925).
[18] DIRAC (1926).
[19] SCHWINGER (1958).

method had been found in quantum mechanics for "the correct treatment of a system of forces propagated with the velocity of light..., of the production of an electromagnetic field by a moving electron, and of the reaction of the field on the electron" (p. 243).[20]

This method was devised by Dirac in his 1927 paper "On the Quantum Theory of Emission and Absorption of Radiation".[20] Following a path initiated by Einstein, he considered the energy E_r and the phase T_r, i.e., the dynamical variable describing the radiation field, as *r* components of the Fourier expansion of the field, and applied the quantisation process to fictitious oscillators.[21] The field plus atom system was described by an Hamiltonian, and the dynamical variables satisfied Hamilton's canonical equations. Then Dirac proceeded to apply the former glorious general transformation theory of the quantum matrices and the symbolisation of the dynamical variables through the previously successful *q* numbers. In his bold approach, he was not discouraged by the fact that, due to the non-invariant Hamiltonian, his treatment was not relativistic. In fact, although he had accounted for the variation of mass with velocity, he treated time as a non relativistic *c* quantity (p. 244).[20] As a mark of success, he finally derived for the radiation absorption and emission coefficients the corresponding expressions of Einstein's theory.

Through Dirac's invention, the radiation field assumed characteristics describable in the complementary particle language. "The ensuing theory of light quantum emission and absorbtion by atomic systems marked the beginning of quantum electrodynamics (QE), as the theory of quantum dynamical system formed by the *em* field in interaction with charged particles" (pp. VII,VIII).[19]

This initial approach to QE was improved by Dirac in his 1930 great work, *The Principles of Quantum Mechanics*, now one of the classics of scientific thought.[22] In compliance with his previous approach, he introduced new mathematical symbols indicating the invariants of the theory (such as the *delta* function, *bra* and *ket* operators, etc). In his Introduction, he mantained that the new physics should abandon the illusion of explaining nature by making "assumptions about the mechanism and forces connecting [the] observable objects", because "her fundamental laws [...] control a substratum of which we cannot form a mental picture without introducing irrelevancies. The formulation of these laws requires the use of the mathematics of transformations". He added that "the substratum which is controlled by the fundamental laws can be properly described only through the invariants":

> The important things in the world appear as the invariants (or more generally the nearly invariants, or quantities with simple transformation properties) of these transformations. The things we are immediately aware of are the relations of these nearly invariants to a certain frame of reference, usually one chosen so as to introduce special simplifying

[20] DIRAC (1927).
[21] EINSTEIN had already shown that, by Fourier expansion, *em* radiation contained in an enclosure, when considered as a classical dynamical system, was equivalent energetically to a denumerably infinite number of harmonic oscillators.
[22] DIRAC (1958).

features which are unimportant from the point of view of general theory (p. VI).[22]

In synthesis, while the method of the coordinates (or method of representations) used set of numbers corresponding to measures of quantities, unimportant in Dirac's new views, his symbolic method dealt directly in an abstract way with the invariants. According to him, this method "seems to go more deeply into the nature of things" (p. VIII):[22] in fact, it made it possible to interpret the former wave and matrix mechanics as distinct forms of representation belonging to the method of the coordinates.

Strange as it may seem, in spite of the fundamental change implied by his symbolic approach, the role of math seemed to Dirac at this time not to be very different from the traditional one:

> From the mathematical side the approach to new theories presents no difficulties, as the math required (at any rate that which is required for the development of physics up to the present) is not essentially different from what has been current for a considerable time. Math is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field. For this reason a book on the new physics, if not purely descriptive of experimental work, must be essentially mathematical. All the same the math is only a tool and *one should learn to hold the physical ideas in one's mind without the reference to the mathematical form* (p. VII, *my Italics*).[22]

However, after a short period, very full of new discoveries, Dirac remarkably modified his former position. In the Introduction to his May 1931 paper, "Quantised Singularities in the Electromagnetic Field", he affirmed:

> There are at present fundamental problems in theoretical physics [...] the solution of which [...] will presumably require a more drastic revision of our fundamental concepts than any that have gone before. Quite likely these changes will be so great that it will be beyond the power of human intelligence to get the necessary ideas by direct attempts to formulate the experimental data in mathematical terms.[23]

In Dirac's views, in the past, "scientific workers" accepted the idea that the progress of physics needed "a math that gets continuously [...] more complicated, but would rest on a permanent basis of axioms and definitions". But this idea was now contradicted by the new physics (and, incidentally, by Dirac's recent achievements), because "modern physical developments have required a math that continuously shifts its foundations and gets more abstract".[23]

He continued by affirming:

> The most powerful method of advance that can be suggested at present is to employ all the resources of pure math in attempts *to perfect and generalise the mathematical formalism* that forms the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities (by a process like Eddington Principle of Identification).[23]

In a few words, in his message to "scientific workers", Dirac wanted to

[23] DIRAC (1931), p. 60

announce that in the future, through an adequate interpretation of its symbols, a new math shall by itself produce a new physics. Explaining in more detail the sense of this interpretation, he mentioned the case of the predicted (his theory) "negative kinetic energy states for the electrons". As it is known, extending Oppenheimer's thesis, he interpreted the negative kinetic energy states in the sense that "*in the world as we know it, all, and not merely nearly all, of the negative energy-states for electrons are occupied*" (italics in text).

Dirac's bold interpretation implied that a rare unoccupied negative state was to be considered as a new particle, the positive electron. Clearly, in this interpretation, a state, formerly representative of the properties of a physical system, was now elevated to the role of the system itself.[24]

5. Conclusions

The fact that through nineteenth century metrology physical laws were written in the form of algebraic equations was considered as an important innovation in the conceptual evolution of physics. In a short time, another innovation was presented in Maxwell's ideas of dimensional formulas, by differentiating the symbols for the operator (representing the system vector, tensor, and so on) from the symbol representing quantities (i.e., measures expressed by rational numbers). I argued that this differentiation of symbols was continued through the quantum mechanical distinction between operators and their eigenvalues up to Dirac's innovative ideas on the math-physics relationship. Dirac's q and c numbers symbolism not only contributed to overcoming the long standing distinction in classical physics between waves and particles in a radiation field, but also resulted in a radical modification in the meaning of symbols, passing from a symbol representing the properties of the system to a mathematical symbol identified with the system itself.

In my view, Dirac's struggle to introduce new symbols paralleled his radical conceptual modification of theory, passing from a form of analysis dealing with properties of a physical system to an analysis of the system itself.

Let us consider that this new status of symbols in physics is to-day tacitly accepted in the structures of elementary particles theories. If a lack of critical comments on its epistemological importance is justified due to the physicists' busy activity, no reason can be found for the silence of philosophers and historians concerning one of the radical modification in the language of modern theories. The fact is that many scholars seem to-day more interested in investigating so-called physical reality and its theoretical description or interpretation. Surely, the relationship between concepts in theory and the presumed corresponding physical objects is worthy of further analysis, and I do not object, in general, to this ontologically oriented historiography. However, I preferred here to call attention to

[24] Max Jammer in his very valuable work presents many reflections on the epistemology of Dirac's contributions, which can be taken as important hints to be developed from the point of view of this paper (pp. 299-301, 308-10, 377-82).[13]

a differently oriented historiography, interested in investigating the tools of our theoretical knowledge, represented by the so-called "material artefacts"[25] or technical components of theories, and, among them, the math of theories. Clearly, I include in this list problems related to the written representations of physics and to the meaning attributed to symbolism all along the historical process.

This historiography remarkably contributed to the debates on physics and on science in the twenties and thirties. Suffice here to mention the representatives of the new empiricists, the neo-Kantian philosophers, and also eminent scientists-philosophers, such as Poincarè, Eddington and Einstein himself, well known for their contributions to the debates on the new sciences of Einstein's relativity and of the rising quantum theories.[26] Although their contributions were variously oriented, it is not difficult to discover that they shared a common interest in the philosophical analysis of the technical language of physics.

In supporting a scientific historiography more epistemologically oriented, I do not intend to exclude the extended areas of other possible approaches, such as those ontologically, sociologically and institutionally oriented.[27] I believe that only a comparison of different approaches can contribute to a better understanding of that momentous phenomenon that we call modern physics.

[25] RENN (1994), p. 5.
[26] HOWARD (1994).
[27] RENN (1994).

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