Swinging the pendulum's interpretation through multimedia[i]

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Abstract

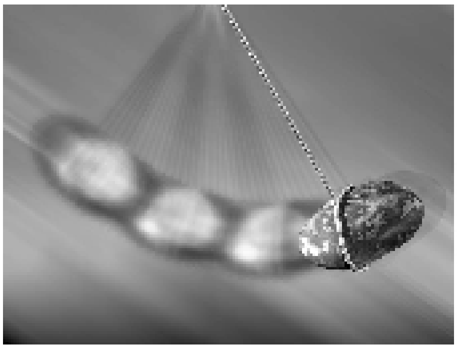
In this presentation a number of animations and simulations are utilized to understand and teach some of the pendulum's interpretations related to what we now see as the history of energy conservation ideas. That is, the accent is not on the pendulum as a time meter but as a constrained fall device, a view that Kuhn refers back to Aristotle. The actors of this story are Galileo, Huygens, Daniel Bernoulli, Mach and Feynman (Leibniz's contributions, however important, are not discussed here). The "phenomenon" dealt with is the swinging body. Galileo, focussing on the heights of descent and ascent rather than on trajectories, interprets the swinging body in both ways (time meter and constrained fall), establishes an analogy between pendulums and inclined planes and eventually gets to the free fall law. Huygens expands the analysis to the compound (physical) pendulum and as a by-product of the search for the centre of oscillation (time meter) formulates a version of the vis viva conservation law (constrained fall). Both Galileo and Huygens assume the impossibility of perpetual motion and Mach's history will later outline and clarify the issues. Daniel Bernoulli generalises Huygens results and formulates for the first time the concept of potential and the related independence of the work done from the trajectories (paths) followed: vis viva conservation at specific positions is now linked with the potential. Feynman's modern way of teaching the subject shows striking similarities.

Multimedia devices enormously increase the possibility of understanding what is a rather physically complex and historically intriguing problem. Teachers and students are in this way introduced to the beauties of epoch-making scientific research and to its epistemological implications.

1. A swinging body and a gestalt switch: constrained fall and isochronism

In Thomas Kuhn's *Structure of Scientific Revolutions* we read:

"Since remote antiquity most people have seen one or another heavy body swinging back and forth on a string or chain until it finally comes to rest."



But did they "see" the same "thing"?

"To the Aristotelians, who believed that a heavy body is moved by its own nature from a higher position to a state of natural rest at a lower one, the swinging body was simply falling with difficulty. Constrained by the chain, it could achieve rest at its low point only after a tortuous motion and a considerable time. Galileo, on the other hand, looking at the swinging body, saw a pendulum, a body that almost succeeded in repeating the same motion over and over again ad infinitum."

Two interpretations are available and here the focus is on the Gestalt switch available to a number of important scientists, our actors, observing a swinging body: the one between constrained fall and isochronism of oscillations.

While at a first reading Kuhn attributes the first to the Aristotelians and the second to Galileo, it is actually well known that the "swinging body" played a main role in Galileo's (and Newton's) interpretations of the fall of bodies.

We propose here to see in the works of Galileo, Huygens and Daniel Bernoulli this capability of "swinging" between the two interpretations of what is, from Galileo on, a "pendulum".

Hoping that teachers and students might eventually do the same, we will tell a story dealing with the less well-known but more ancient interpretation of the swinging body: the one that still sees it as a constrained fall (even if in a vacuum). It will deliver a number of unexpected goods and introduce us to the concept of "potential" and eventually to the interplay between "actual" and "potential" "energy" in the principle of energy conservation. It is no wonder there is a claim that this tradition started with Aristotle (and in "modern" times continued through Leibniz).

2. Galileo: equal heights of ascent and descent; the law of free fall

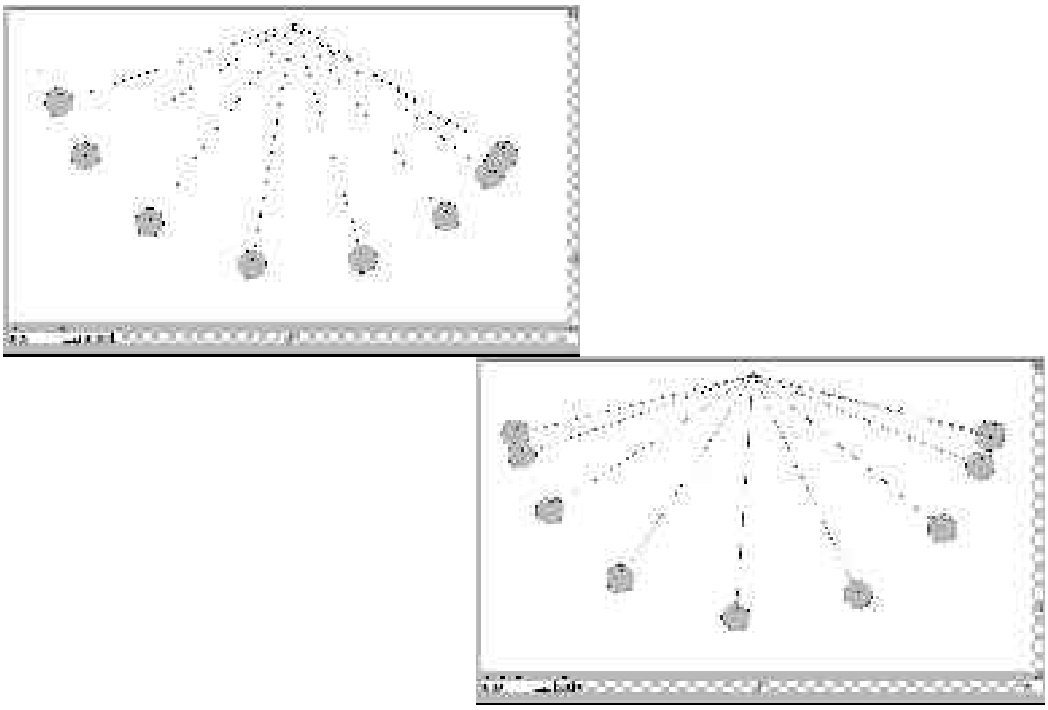
The basic assumption Galileo makes observing the swinging body appears at an early stage of his career. In fact studying the constrained fall on an inclined plane Galileo in his "*De motu*" (*On Motion*), written between 1589 and 1592, asserts that:

"[…] a heavy body tends downward with as much force as it is necessary to lift it up"

This is a shift of attention, from the actual movement and the actual trajectory of the body to the height of descent and of ascent. In 1638 a full and mature expression is found in the *Discorsi* (First Day):

"As may be clearly seen in the case of a rather heavy pendulum which, when pulled aside fifty or sixty degrees from the vertical, will acquire precisely that speed and force (virtù) which are sufficient to carry it to an equal elevation save only that small portion which it loses through friction on the air."

Certainly this shift of attention to the height of descent and of ascent in the constrained fall was not an easy step. Obviously it was not an observation: the pendulum in standard conditions does not rise to the same height of descent, as can be seen also through a computer simulation:



Simulation of pendulum motion with and without air

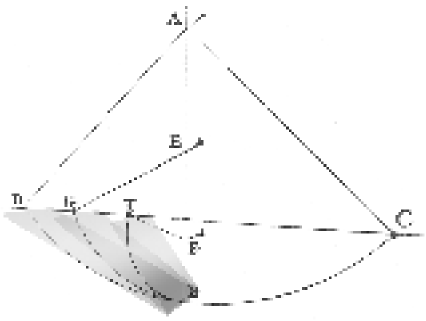
But it is easy today to show what Galileo had in mind: removing “impediments” such as air resistance, the pendulum actually oscillates in agreement with Galileo’s assumption.

The main assumption of the third day of the *Discorsi* is that:

“The speeds acquired by one and the same body moving down planes of different inclinations are equal when the heights of these planes are equal.”

This means that the final velocity of fall depends on the vertical height of descent (elevation) and not on the trajectories actually followed (inclinations).

A correspondence is established between pendulum trajectories and planes with differing inclinations. It is now in fact a “constrained” pendulum motion that “establishes” the assumption: the accent is not on the trajectories but on the initial and final heights:



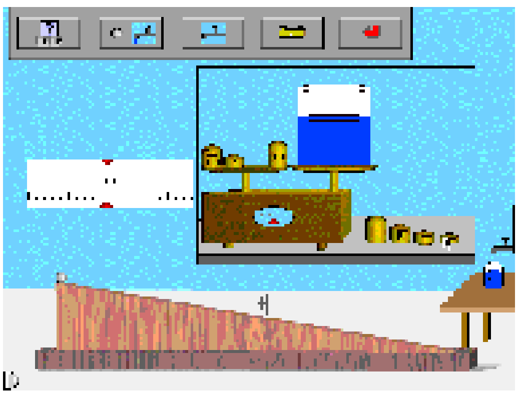
Here again a reconstruction and an animation can help us “see” that when the fall of the pendulum is constrained by nails fixed on the vertical, whenever possible the weight rises at the same height, even if not in a symmetrical position, and when even that becomes impossible (the nail is in such a position that the length of the string left free is too short) it shows it still has a capacity of movement that makes it revolve around the "impediment".

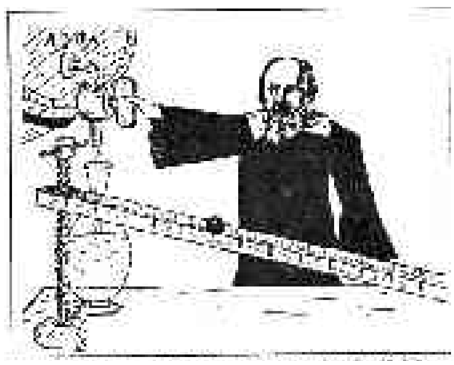
But what lies behind Galileo's assumption? The principle, already expressed long before by Leonardo, that bodies cannot be raised to a higher level uniquely by virtue of their own weight, an early statement of the principle of impossibility of perpetual motion. The bob of the pendulum in its periodical idealised motion cannot rise at a higher or lower level than that of the first descent; otherwise work would be produced out of nothing.

The important quantity connected with the initial and final height is thus the velocity acquired during the fall. To every height of fall corresponds a final velocity acquired during the fall. A link between a static, positional, quantity (height) and a cinematic one (velocity) is pointed to.

Which is the mathematical relation that connects the two?

A simulation can simplify the understanding of Galileo's procedures in the famous passage of the *Discorsi*:





The height of fall plays a double role here, in the initial position of the ball on the inclined plane and in the level of the water in the water vessel: the water vessel is large and the tube, leading to the tap, thin so that the height of fall of the water is basically constant. This, assuming a relation between the height of the level of the water in the vessel and the velocity of the falling water through the tap, allows a constant flux of water and thus the precision of the time measurement.

The results of Galileo's efforts can be summarised in modern terms as follows (g is the gravity acceleration, $a=g\sin\theta$ is the acceleration that varies with inclination, $s=h/\sin\theta$ is the length of the inclined plane, h its height)

- the instantaneous velocity is proportional to the time elapsed: $v=at$
- space is proportional to square times: $s=at^2/2$
- from a) and b) we get: $s=v^2/2a$,

that is:

- the final velocity is proportional to the square root of the height $v_f=\sqrt{2gh}$

This is a basic law because it connects, perhaps for the first time, position and velocities, statics and kinematics. One of the extraordinary features of this law, lost in modern textbooks, is that the two quantities, position and velocity, are not taken at the same instant. The velocity is the one that the body acquires falling from the height, that is, it is the "virtual" or "potential" velocity that it would acquire if it fell from that height. Reversing the two we can also say that a body with such a velocity can raise itself to such a position. The pendulum thus acquires a new meaning: in the first quarter of period the weight falling acquires a velocity that, without friction and other impediments, will raise it on the symmetrical side in the second quarter to the same height of descent. The same happens in the third and fourth quarters till the weight reacquires its original height.

3. Huygens: from the centre of oscillation of a compound pendulum to vis viva conservation at specific positions

In 1673 Christiaan Huygens in his *Horologium Oscillatorium* gives a relevant contribution to our story solving a difficult and important problem. Pendulums in nature are not ideal objects, but real ones with weights that are not concentrated in a point at the end of a weightless string. In the context of his time measuring efforts, Huygens needed an answer to the question: what is the centre of oscillation of a compound pendulum? That is: what is the length of a simple pendulum that oscillates with the same period of the compound pendulum given? The search for the solution of this problem, perhaps the greatest among his many achievements, produced important results also for our story of the constrained fall. Huygens' early attempts date back to 1661 and 1664 but we refer here to the 1673 account.

Huygens, generalising Galileo's approach, formulates two basic assumptions and a number of propositions. While Galileo was concerned with a single body, Huygens deals with a number of them, that is with a system of connected bodies, and thus his concern is with their centre of gravity. The first hypothesis asserts that

"If any number of weights begin to move by the force of their own gravity, their centre of gravity cannot rise higher than the place at which it was located at the beginning of the motion".

This statement in its apparent simplicity will have the most extraordinary consequences. It is no wonder then that Huygens makes an effort to explain its meaning. He actually states here, and a number of times after, that the real content of the hypothesis is simply that bodies cannot "uniquely by virtue of their own weight" rise at a height higher than the one of fall, a statement that, he asserts, is largely shared.

But now Huygens introduces a comment that was implicit in Galileo's *Discorsi*:

"Indeed, if those builders of new machines who tried in vain to produce perpetual motion [motum perpetuum] had known how to use this hypothesis, they would have easily seen their errors and would have understood that this is in no way possible through mechanical means [mechanica ratione]"

In modern terms: a quantity of work cannot be produced without a corresponding compensation, perpetual motion is impossible.

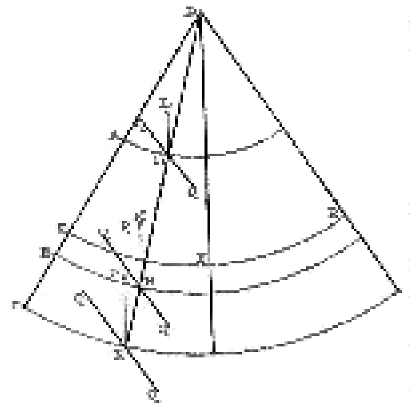
A second hypothesis follows:

"Air and any other manifest impediment having been removed, as we wish to be understood in the following demonstrations, the center of gravity of a rotating pendulum crosses through equal arcs in descending and in ascending"

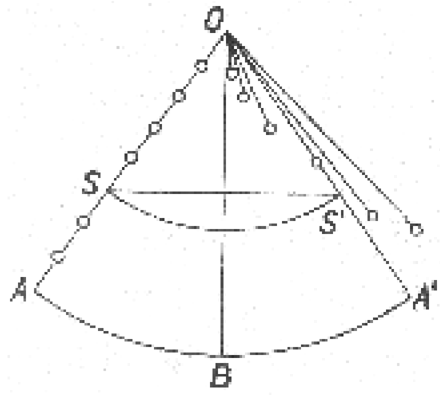
In fact one cannot imagine a pendulum that after each two quarters of period rises "uniquely by virtue of its own weight" to a higher position! But Huygens' great achievement here is to apply this principle to the centre of gravity of the compound pendulum. From this extension of a Galilean line of thought extraordinary consequences will follow.

Huygens, in proposition III specifies that $H = \Sigma m_i r_i / \Sigma m_i$ (where H is the height of ascent-descent of the center of gravity, m_i are the weights and r_i their heights of ascent-descent), and in proposition IV states that the removal of the constraints between the bodies or parts of the bodies does not influence the equivalence between height of ascent and descent. In modern words: these constraints do not perform work.

"Assume that a pendulum is composed of many weights and, beginning from rest, has completed any part of its whole oscillation. Imagine next that the common bond between the weights has been broken and that each weight converts its acquired velocity upwards and rises as high as it can. Granting all this, the common centre of gravity will return to the same height which it had before the oscillation began."



Huygens



Mach

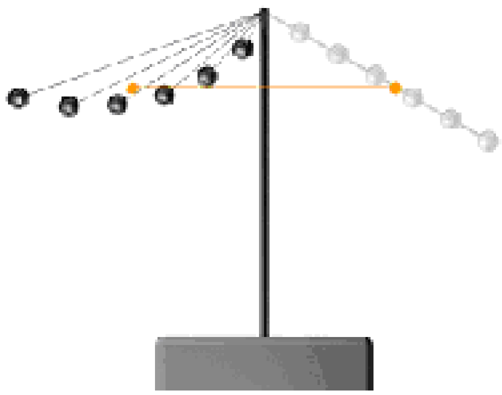
4. Mach's version

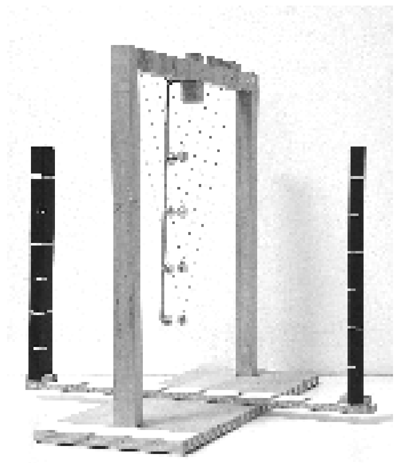
We follow here, for pedagogical purposes, Mach's version of this important formulation: the centre of gravity regains its initial height not only after a free fall and a free ascent, not only after a constrained fall and a constrained ascent, but also after a constrained fall and a free ascent.

If the constraints are removed at the end of the first quarter of period on the vertical line, the pendulum's weights can move freely in the second quarter with the initial velocities equal to the final velocities of constrained fall acquired. Some of the weights will rise at a lower level than in the case of free fall and some to a higher level. If we manage to record the maximum height of ascent of the single weights (due to differing lengths of the pendulums they will be reached in differing times) we could calculate the position of the centre of gravity and understand that Huygens' hypotheses are correct.

How to remove the constraints without perturbing the motions?

We imagine that on the vertical line at the end of the first quarter of period a compound pendulum made by weights (iron balls or marbles) connected by a weightless constraint (balsa wood) hits an equal number of equal weights individually suspended and thus free to move. Assuming the conservation of momentum in the impact we see that the weights rise to different heights, higher or lower, in agreement with Huygens' statements. We also realise the difficulty of a precise experimental assessment of the hypothesis.





A visualization can be achieved through a computer animation

The automatic removal of the constraints, the tracking that shows the different maximum heights of ascent, and the automatic calculation of the position of the centre of gravity are of great help not in proving the hypotheses (the software is build around the mechanical laws we are dealing with) but in understanding them.

As to free descent, according to Galileo's result, the vertical distance covered by a heavy body in free fall starting from rest is proportional to the square of the velocity acquired in the fall, with which velocity it could rise to the same height. Applying the relation $v_f = \sqrt{2gh}$ to each free falling weight:

$$\Sigma m_i r_i = \Sigma m_i v_i^2/2g$$

In the case of the constrained descent we can detect the individual velocities only through the individual heights of free ascent (experimentally found) in the second quarter of period. But now we can apply Galileo's relation to the heights of ascent and express the final velocities of the constrained fall with the same law (the letter u is used to indicate velocities acquired in the constrained fall):

$$\Sigma m_i r'_i = \Sigma m_i u_i^2/2g$$

Thus the result of the equivalence of the height of ascent and descent of the centre of gravity is:

$$\Sigma m_i v_i^2/2g\Sigma m_i = \Sigma m_i u_i^2/2g\Sigma m_i$$

and thus: $$\Sigma m_i v_i^2 = \Sigma m_i u_i^2$$

Thus Huygens' result consists in this: for a system of bodies under the effect of gravity, the sum of the products of the masses multiplied by the squares of the final velocities is the same, whether the bodies move constrained together or whether they move freely from the same vertical height. It appears from this result that $\Sigma m_i v_i^2$ is an important quantity, which is characteristic of the position of the system (its vertical height) and does not depend on the paths followed to get to that position, under the assumed conditions. Again we have to remember that in this quantity, characteristic of a system in a given position, the velocities, whether constrained or free, are the final velocities of the "virtual" or "potential" fall.

Thus the (compound) pendulum has delivered a very good result: it helped identify one very important physical quantity, to be called "vis viva", the "modern" capacity of a body to perform "work", its dependence on the position of the system of bodies and its independence from those constraints which do not perform "work". Returning back to the initial position (completing a closed path) the value of the vis viva does not change, it does not depend on the actual trajectories: it is a constant of the system for a given position. Here this is the meaning of “conservation of vis viva”. In fact the “vis viva” during motion varies at each instant, given the variation of the actual velocities.

5. Bernoulli and the birth of the potential

In his *Hydrodynamica* of 1738 Daniel Bernoulli (the first to introduce the potential function) discusses at length the relations between “descensus actualis” and “ascensus potentialis”

In his approach of 1748 to our theme the starting point is the conservation of vis viva derived from Huygens’ results:

$$mv^2 + m'v'^2 + m''v''^2 + ... = mu^2 + m'u'^2 + m''u''^2 + ...$$

How can this law be utilized to connect velocities and external forces? Through Galileo’s theorem: in fact, in the case of uniform and parallel gravity, the square of the velocity gained is proportional to the displacement and since this is independent from the path of the body: “there is always conservation of vis viva with respect to the height from which the fall takes place”.

Assuming the acceleration of gravity as equal to unity and the vertical fall distances equal to x, x’ and so on:

$$u^2 = 2x,\ u'^2 = 2x',\ u''^2 = 2x'', ...$$

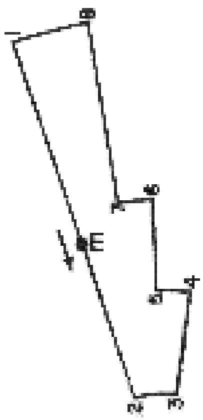
the expression of conservation of vis viva becomes

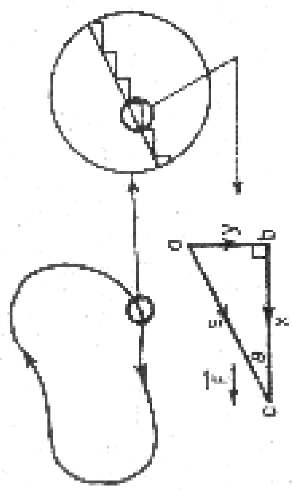
$$mv^2 + m'v'^2 + m''v''^2 + ... = 2mx + 2m'x' + 2m''x'' + ...$$

and thus “the total vis viva is equal to the product of the total mass of the system with twice the vertical distance the centre falls”.

From a modern point of view, the second member expresses double the “work” done by the forces acting on the system (in this case central forces and unity of acceleration): force (mass time acceleration) time distance. The vis viva of the system in a certain position (velocities are here still the final velocities of a potential fall) equals the “work” done to get to that position or the capacity to do “work” falling from that position.

The variation of vis viva depends on the distance and not on the trajectory: vis viva at D and C is the same, moving from C to D there is no change of vis viva (no “work” is done along paths perpendicular to the force, the difference in vis viva between A and D thus does not depend on the trajectory followed, straight down from A to D or through C). Vis viva at D depends only on the distance to the centre of attraction, it is now a positional quantity. In the closed path DACD there is no gain or loss of “work”. The difference of “work” depends only on the initial or final positions and not on the path. The “positional” vis viva is thus an indication of potential “work”, later to be called potential energy. The variation of the vis viva is equal to the variation of the potential “work”.





D.Bernoulli R.Feynman

6. Modern textbooks still utilize Bernoulli's approach

Feynman's 1963 *Lectures in Physics*, is revealing. In discussing work done by gravity, Feynman wants to show that the total work done in going around a complete cycle is zero, in agreement with the impossibility of perpetual motion.

He thus analyses a closed path in a gravitational field and shows that on the circular paths the work is zero because the force is at right angles to the curve, and on the radial paths the total work is again zero because it is the sum of the same amount of work done once in the direction of the centre of attraction and the second in the opposite one.

Is the situation different for a real curve? No, because we can refer back to the same analysis: the work done in going from *a* to *b* and *b* to *c* on a triangle is the same as the work done in going directly from *a* to *c*. In the same chapter Feynman, dealing with planetary motion, asserts that:

"So long as we come back to the same distance, the kinetic energy will be the same. So whether the motion is the real undisturbed one, or is changed in direction by channels, by frictionless constraints, the kinetic energy with which the planet arrives at a point will be the same."

A clear, even if implicit and perhaps unaware, reference to Daniel Bernoulli's results (through the mediation of the tradition of rational mechanics): work only depends on the initial and final positions (difference of potential) and not on the actual path (trajectory).

Thus the insight that pendulums without impediments can only rise back to their original heights has produced, through a number of achievements, a very important and lasting historical result: from vis viva conservation at specific positions we get the concept of potential, a remarkable gestalt switch from isochronism and a big step towards what is now energy conservation.

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