

H. HERTZ'S EXPERIMENTS AND THE SHIFT TOWARDS CONTIGUOUS PROPAGATION IN THE EARLY NINETIES

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"But in no way can a direct proof of
these equations be deduced from
experience"

Hertz 1888

The history of C.E.T. (Classical Electromagnetic Theory) deserves great attention. A basic problem is still open: why in Lorentz's synthesis, the model of contiguous propagation of electromagnetic interactions was preferred to delayed action-at-a-distance. Both approaches in fact explained the time delay and, as is widely known, gave results that were formally equivalent. But at the beginning of the nineties a number of leading Continental scientists had shifted towards contiguous propagation, including Helmholtz, Hertz, Planck, Lorentz and Poincarè. They did not accept the whole of Maxwell's theory: Maxwell's electromagnetic continuum was rejected, but contiguous action was kept. The usual answer to this problem is that Hertz's experiments (1888-1890) played a crucial role in favour of contiguous propagation. The aim of this paper is to show that Hertz's experiment were not crucial at all: evidence is given that Hertz himself did not shift towards contiguous propagation from his previous action-at-a-distance approach for experimental reasons and that Poincarè asserted the cruciality of the experiments only from 1894, but changed his judgement again in 1902.

Evidence for my interpretation is found in the well-known second volume of Hertz's collected papers: "Electric Waves". This book consists of an experimental and a theoretical introduction, written in 1892, and of a collection of electromagnetic papers published from 1887 to 1890. These papers can be divided into four groups aiming at:

- a) an experimental proof of electromagnetic effects in insulators: (“On very rapid electric oscillations”; “On the action of a rectilinear electric oscillation upon a neighbouring circuit”; “On electromagnetic effects produced by Electrical Disturbances in Insulators”; 1887-8)
- b) an experimental proof of the finite propagation of interactions and the existence of waves in air: (“On the Finite Velocity of propagation of electromagnetic actions”; “On electromagnetic waves in air and their reflection”; 1888)
- c) an experimental proof of analogous behaviour of light and electromagnetic radiation: (“The propagation of electric waves by means of wires”; “On electric radiation”; “On the mechanical action of electric waves in wires”; 1888-91)
- d) a theoretical exposition of Hertz’s interpretation: (“The forces of electric oscillations, treated according to Maxwell’s theory”, written in 1888, published in 1889; “On the fundamental equations of electromagnetics for bodies at rest”; “On the fundamental equations of electromagnetics for bodies in motion” (1890).

In order to understand the development of Hertz’s ideas and experiments it is useful to recall the schemes that Hertz himself described in the theoretical part of the Introduction to "Electric Waves". Hertz distinguishes four conceptual models: action-at-a-distance, potential theory, action-at-a-distance with an ether, contiguous propagation. The most important models in the eighties were the third and the fourth. The third represented Helmholtz’s theory. Two limiting cases (IIIa and IIIb) of this theory are particularly relevant.

Both refer to a conception of action-at-a-distance starting from the charges on the plates that polarize the dielectric between the plates. In the first case, the energy is supposed to be concentrated on the plates and the actions (at a distance) produced by the polarised

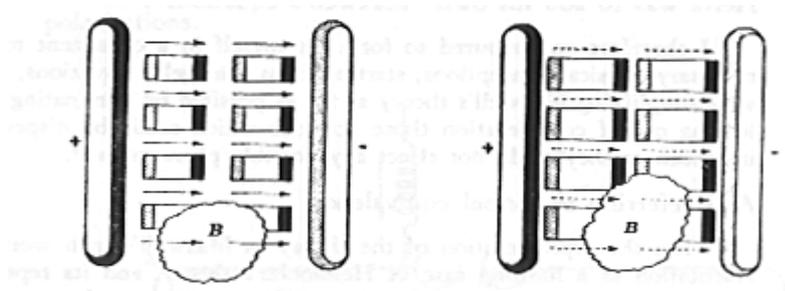


Fig.1(Source: Hertz (1893) pp. 24-25

dielectric are comparatively small. Ideally, removing B leaves some action-at-a-distance. In the second case, the energy is supposed to be concentrated in the dielectric particles and so the dielectric action is the prevailing one. This second case is mathematically equivalent to Maxwell's. But in both cases the charges are the sources both of action-at-a-distance effects and the effects of the polarisations of ether. These polarisations in turn were supposed to produce other action-at-a-distance effects. The fourth model is radically different: the polarisation and not the charges are supposed to be the primary sources of action. All the energy is localised in the medium, no action-at-a-distance exists.

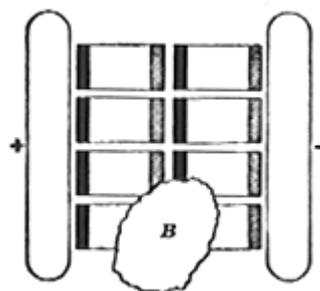


Fig. 2 (Source: Hertz (1893))

Now the problem is: how can Maxwell's Treatise and Hertz's researches be framed in these models? As far as Maxwell is concerned, Hertz's analysis is very clear: Maxwell aimed at case IV but stopped between IIIb and IV. He did not take the final step, which was later accomplished by Poynting. The problem now is to place Hertz's papers from 1887 to 1890 in his own classification of 1893. There is no doubt that the two theoretical papers published in 1890 (group d above) belong to the fourth model. But was this fourth model the starting point of Hertz's researches? Of course not. Hertz himself acknowledges that his own starting point was Helmholtz's theory, i.e. the third model. In fact he asserts that:

"Notwithstanding the greatest admiration for Maxwell's mathematical conceptions I have not always felt quite certain of having grasped the physical significance of his statements. Hence it was not possible for me to be guided in my experiments directly by Maxwell's book. I have rather been guided by Helmholtz's work, as indeed may plainly be seen from the manner in which the experiments are set forth."

The problem Helmholtz proposed in 1879 to Hertz was the following:

"To establish experimentally any relation between electromagnetic forces and the dielectric polarisation of insulators - that is to say, either an electromagnetic force exerted by polarisations in nonconductors, or the polarisation of a non-conductor as an effect of electromagnetic induction."

This problem was faced in 1886 by Hertz in its original theoretical framework: Helmholtz's theory of action-at-a-distance with dielectric. The problem was not treated as a direct proof of Maxwell's displacement current nor of contiguous propagation. It was treated as a problem of selecting between IIIa and IIIb. This is the true original meaning of Hertz's first two series of experiments: the corroboration of IIIb.

In fact Hertz asserts that the problem has been propounded in the following connection:

"If we start from the electromagnetic laws which in 1879 enjoyed universal recognition, and make certain further assumptions, we arrive at the equations of Maxwell's theory which at that time (in Germany) were by no means universally recognised."

The assumptions to shift from IIIa to IIIb are the following:

"First, that changes of dielectric polarisation in non-conductors produce the same electromagnetic forces as do the currents which are equivalent to them; secondly that electromagnetic forces as well as electrostatic are able to produce dielectric polarisations; thirdly, that in all these respects air and empty space behave like all other dielectrics".

These assumptions had actually been made by Helmholtz in 1870 to get to IIIb:

"In the latter part of his paper ('On the Equations of Motion of Electricity for conducting Bodies at Rest') von Helmholtz has deduced Maxwell's equations from the older views and from hypotheses which are equivalent to those just stated."

Thus Hertz's problem was to prove these assumptions:

"The problem of proving all three hypotheses, and thereby establishing the correctness of the whole of Maxwell's theory, appeared to be an unreasonable demand; the Academy therefore, contented itself with requiring a confirmation of one of the first two".

In the previous quotations "Maxwell's theory" has to be interpreted as: "the limiting case IIIb of Helmholtz's theory". Helmholtz too refers to "Maxwell's theory" with the same meaning. In fact as late as 1894:

"There were (in Maxwell's theory) no longer any unclosed currents, for the accumulation of electric charges at the ends of a conductor, and the simultaneous dielectric polarisation of the medium between them, represented an equivalent electric motion in the intervening dielectric, thus completing the gap in the circuit".

Here three elements contradict model IV: charges as sources of polarisations, instantaneous action-at-a-distance ("simultaneous polarisation"), electric charge as an incompressible fluid.

Thus a clear distinction has to be kept in mind between equivalence of formal results and differences of conceptual models. Models IIIb and IV are radically different from the conceptual point of view, but formally they are equivalent: in 1886 the equivalence was between Helmholtz's limiting case ($K=1$) and Maxwell's equations, in 1890 Hertz was to add his own "Maxwell's equations":

"I therefore endeavoured to form for myself in a consistent manner the necessary physical conceptions, starting from Maxwell's equations, but otherwise simplifying Maxwell's theory as far as possible by eliminating or simply leaving out of consideration those portions which could be dispensed with, inasmuch as they could not affect any possible phenomena."

And referring to formal equivalence:

"Thus the representation of the theory in Maxwell's own work, its representation as a limiting case of Helmholtz's theory, and its representation in the present dissertations - however different in form - have substantially the same inner significance ... To the question, 'What is Maxwell's theory?' I know of no shorter or more definite answer than the following: Maxwell's theory is Maxwell's system of equations. Every theory which leads to the same system of equations, and therefore comprises the same possible phenomena, I would consider as being a form or special case of Maxwell's theory."

So far we have reached the following conclusions: there was a difference of conceptual models but a formal equivalence between IIIb and IV; Hertz started his researches to choose between IIIa and IIIb (two limiting cases of Helmholtz's theory).

In the first group of papers there is an account of how Hertz in 1886 started to attack the first step of the problem of 1879; i.e. whether the changes of dielectric polarisation in nonconductors produce the same electromagnetic forces as do the currents which are equivalent to them. In fact Hertz had to face several problems mainly connected with the weakness of the supposed inductive effect generated by an alternating polarisation of a dielectric. A faster alternation would have increased the effect (faster variation of magnetic force) as well as a greater force applied. Thus Hertz started experimenting with oscillating discharges on open circuits. The open extremities were large metallic surfaces: an interposed block of dielectric would have periodically changed polarisation because of the effect of the alternate electric action. The inductive resulting effects with and without dielectric could be compared to reveal the inductive effect of polarisations.

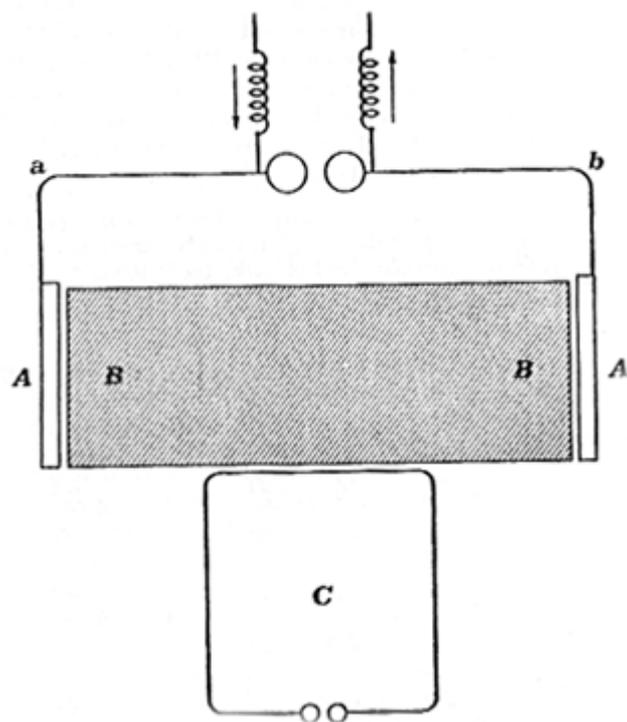


Fig. 3 (Source: Hertz (1893) p. 5)

Despite the ingenious invention of the spark excitator and spark resonator of the very fast oscillations, the first experiments did not lead to interesting results. Hertz, working in Helmholtz's framework, was puzzled by the electrostatic effects: he wanted to detect only the electromagnetic ones (in Helmholtz's theory the two are not unified). A solution of the problem required great care and was found in 1887 by means of this apparatus:

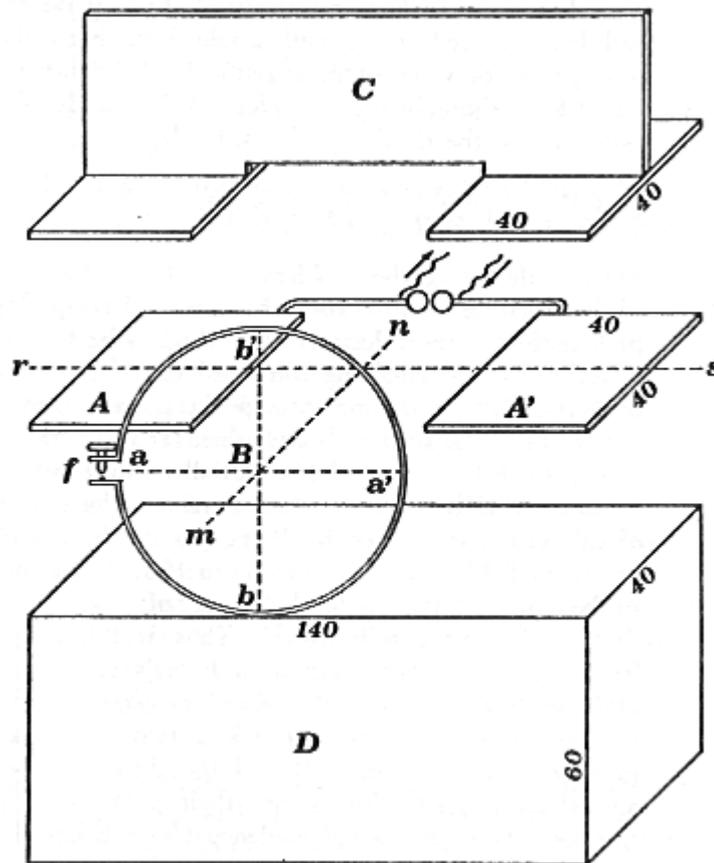


Fig. 4 (Source: Hertz (1893) p. 97)

where A is the excitator, B the resonator, C a conductor and D a dielectric.

Thus the first assumption of the three was positively proved and Hertz obtained the Academy's prize. Hertz started working on the second hypothesis, i.e. "that electromagnetic forces as well as electrostatic are able to produce dielectric polarisations". But suddenly he realised that the case IIIb would have been better tested through the third hypothesis, i.e. "that in all these respects air and empty space behave

like all other dielectrics". Even if the two first hypotheses had been proved, there still would have been the possibility of a propagation of wave of polarization in dielectrics with a velocity different from the light velocity. Hertz, still in a Helmholtzian framework, shifts to test IIIb, for he

"felt that the third hypothesis contained the gist and special significance of Faraday's, and therefore of Maxwell's view."

Thus at this stage the problem is to demonstrate "in air a finite rate of propagation and waves". Moreover the equality of this rate of propagation to the velocity of light had to be tested. Hertz again used a device with which he compared the effects in conductors and in dielectrics (air); this time through the phenomenon of interference. He obtained a different, but finite value, for the two cases.

In 1892 he explained his partially wrong results (from the point of view of IIIb the two values have to coincide) as caused by errors of calculation (remarked by Poincarè) and by the effects of the walls of his small laboratory. But still in 1887-'8 he considered the results to be correct. The same difference of velocities resulted after the discovery of wave reflections. Thus at this stage (March 1888) at least one main aspect existed in Hertz's researches that contradicted IIIb: the difference of velocities of propagation of electromagnetic effects in wires and in air, and at least two aspects contradicted IV: the previous one (experimental) and the difference between electrostatic and electromagnetic forces and their velocities (theoretical). Despite the contrary experimental evidence Hertz believed in the equality of velocities. It has to be remarked that Hertz knew of the experimental confirmation of his own beliefs only after he wrote (but before he published) the 1893 Introduction. In fact he acknowledges the results of Sarrasin and de La Rive in a note, while in the text he still refers to an existing difference of velocities.

This is the first historical evidence against experimental bases for the shift towards contiguous propagation: in this theory only one kind of force and one velocity

(light velocity) is admitted both for wires and for air. But despite his own experimental results, partially contrary to IIIb and a fortiori contrary to IV, Hertz shifts at this stage towards model IV. In his first theoretical paper (of 1888), he reinterprets the previous experiments from the point of view of Maxwell's theory; here Maxwell's theory means model IV and no longer IIIb. The first pages of this paper are extremely interesting. They show that finite propagation of interactions and waves can be explained in Helmholtz's theory (IIIb), but that the explanation is more complex than in the "pure development" of Maxwell's theory (IV):

"... I based my first interpretation of these experiments upon the older views, seeking partly to explain the phenomena as resulting from the cooperation of electrostatic and electromagnetic forces".

But: "To Maxwell's theory in its pure development such a distinction is foreign. Hence I now wish to show that the phenomena can be explained in terms of Maxwell's theory without introducing this distinction."

$$\begin{array}{l}
 \text{1) } A \frac{dL}{dt} = \frac{dZ}{dy} - \frac{dY}{dz} \\
 A \frac{dM}{dt} = \frac{dX}{dz} - \frac{dZ}{dx} \\
 A \frac{dN}{dt} = \frac{dY}{dx} - \frac{dX}{dy} \\
 \text{2) } A \frac{dX}{dt} = \frac{dM}{dz} - \frac{dN}{dy} \\
 A \frac{dY}{dt} = \frac{dN}{dx} - \frac{dL}{dz} \\
 A \frac{dZ}{dt} = \frac{dL}{dy} - \frac{dM}{dx} \\
 \text{3) } \frac{dL}{dx} + \frac{dM}{dy} + \frac{dN}{dz} = 0 \quad \text{and} \quad \frac{dX}{dx} + \frac{dY}{dy} + \frac{dZ}{dz} = 0
 \end{array}$$

Thus Hertz writes the Maxwell equations in the following form:

where X, Y, Z are the component of the electric force and M, N, L of the magnetic one, A is the reciprocal of light velocity and Gaussian units are utilized. Then Hertz states the energy values. The values are the same as in Maxwell but no mechanical interpretation of potential and kinetic energy is given to the electric and magnetic energy:

$$\frac{1}{8\pi} \int (X^2 + Y^2 + Z^2) d\tau ; \frac{1}{8\pi} \int (L^2 + M^2 + N^2) d\tau .$$

No doubt this is done on purpose, for the final shift to contiguous propagation, that is from IIIb to IV, is taking place:

"These statements form as far as the ether is concerned, the essential parts of Maxwell's theory. Maxwell arrived at them by starting with the idea of action-at-a-distance and attributing to the ether the properties of a highly polarisable dielectric medium. We can also arrive at them in other ways. But in no way can a direct proof of these equations be deduced from experience. It appears most logical, therefore, to regard them independently of the way in which they have been arrived at, to consider them as hypothetical assumptions, and to let their probability depend upon the very large number of natural laws which they embrace. If we take up this point of view we can dispense with a number of auxiliary ideas which render the understanding of Maxwell's theory more difficult, partly for no other reason than that they really possess no meaning, if we finally exclude the notion of direct action-at-a-distance."

This quotation shows Hertz's clear ideas about scientific methodology: no direct proof of these equations can be "deduced from experience". He was perfectly aware that the shift was theoretical, as well as the fact that energy conservation in a framework of contiguous propagation was no longer the sum of potential and kinetic energy but was expressed by a continuity theorem of local conservation. In fact in the following lines he reports Poynting's theorem:

$$\begin{aligned} \frac{d}{dt} \left\{ \frac{1}{8\pi} \int (X^2 + Y^2 + Z^2) d\tau + \frac{1}{8\pi} \int (L^2 + M^2 + N^2) d\tau \right\} = \\ = \frac{1}{4\pi A} \int \{ (NY - MZ) \cos(n, x) + (LZ - NX) \cos(n, y) + \\ + (MX - LY) \cos(n, z) \} d\omega \end{aligned}$$

where (n,x) , (n,y) , (n,z) denote the angles which the normals from dl make with the axis.

"This equation shows that the amount which the energy of the space has increased can be regarded as having entered through the elements of the surface."

Case IV is finally achieved. All the energy is in the medium, the polarisation coincides with the force, the dielectric constant of the ether is dropped, electricity is no longer an incompressible fluid, local conservation of energy is reached

At this stage (1888), Hertz starts his third series of researches, leading to an experimental corroboration of case IV, that is, of Maxwell's electromagnetic theory of light. Electromagnetic radiation was shown to possess properties analogous to light (reflection, refraction, diffraction, etc.) These experiments attracted great attention. Hertz is perfectly aware that a considerable part of the attention devoted to his experiments is "of philosophical nature" and it is connected with the philosophical debate about the kinds of action. Again in 1892 he honestly acknowledges the main scientific result of his researches:

"Casting now a glance backwards we see that by the experiments above sketched the propagation in time of a supposed action-at-a-distance is for the first time proved. This fact forms the philosophic result of the experiments; and, indeed, in a certain sense the most important result."

The propagation in time, already predicted in the sixties by all the different theories, is finally corroborated experimentally. All the other results are considered by Hertz as highly probable, but:

"What we here indicate as having been accomplished by the experiments is accomplished independently of the correctness of particular theories."

At the same time, Hertz expresses his preference for Maxwell's theory in the sense IV that is, for the reinterpretation of Maxwell's theory he had carried out in the last two theoretical papers of the fourth group of his researches (1890). Thus he finally asserts that "in this connection" the best characterisation of his experiments is to confirm the fundamental hypotheses of the theory of Faraday-Maxwell.

In my opinion, Hertz gives a correct account of the theory-experiment connection in his researches: no crucial experiment exists, no direct relation from experiments to theory. But a theoretical preference for a specific point of view has heuristic power (the third group of experiments were led to by the analogy suggested by the electromagnetic theory of light) and allows a good interpretation of all the results.

There are reasons of simplicity in favour of contiguous propagation: reduction of the number of forces, velocities, unification of polarisation and forces, local conservation. Hertz does not differentiate the idea of contiguous propagation from the idea of a substantial ether. For him, contiguous propagation is the action through a medium and not the action whose effects on a point depend only on what happens at the vicinity of the point itself. There is no doubt that in this context the experimental proof of the time delay is interpreted as the proof of the existence of an ether.

Support for my interpretation that the shift towards contiguous propagation was not due to Hertz's experiments is given by Poincaré's analysis of Hertz's results. Up to 1904 Poincaré did not consider these experiments as crucial in favour of contiguous propagation, but again in 1902 he points the difficulties of their's interpretation.

The first edition of the first volume of Poincaré's *Electricité et Optique* was published in 1890. It collects the lectures given between March and June 1888 (and not 1889 as erroneously printed on the volume). The whole book concerns Maxwell's theory. Little room is left for Hertz's experiments. At the end of his introduction, Poincaré asserts that his assistant, M. Blondin, had prepared a chapter on Hertz's experiments, but that he himself preferred to postpone this analysis to the second volume of the work. So in 1890 Hertz's experiences were not analysed by Poincaré, despite his assertion that since 1888 Maxwell's theory had received very clear experimental confirmation. At the end of the book Poincaré asserts:

"La théorie est incomplète, les expériences sont peu nombreuses et contradictoires. Il est donc impossible de décider s'il y a accord ou désaccord. Je termine encore par un point d'interrogation. Toutefois, s'il m'est défendu de conclure, je puis parler de l'impression qui me causent les plus récents progrès de la science, et

que le lecteur partagera sans doute après lu ces notes. Cette impression encore bien vague est que l'ensemble des résultats est plus favorable aujourd'hui à la théorie de Maxwell qu'il y a quelques mois au moment où j'ai clos mon cours."

In Poincaré's second volume, published in 1891 and containing the lectures of March-June 1890, Hertz's experiments find a place in a chapter written by M. Blondin. But in the Preface Poincaré notes that: "Beaucoup de personnes trouveront cette tentative bien prématurée et elles n'auront pas tort; je n'ai pu arriver à aucune conclusion définitive, les résultats expérimentaux ne le permettent pas encore."

Thus in 1890, despite a short note that between June and November Poincaré changed his mind on "certain points" due to the experiments of Sarrasin and de la Rive, Hertz's experiments were not considered crucial at all.

In 1894 Poincaré published a careful analysis of Hertz's theoretical and experimental work. The division of the experiments is the standard one: fast oscillations, propagation in wires, propagation in air, in dielectrics different from air. The delay in propagation in wires is considered a consequence of both Maxwell's and Kirchhoff's theories. The crucial point is considered here to be the establishment of a delay in propagation in air. This delay being equal to that of the conduction in wires, and both being dependent on the velocity of light, Maxwell's theory is supposed to be confirmed. Later, Hertz's experimentally found analogies with optical properties were to lend further support to Maxwell's electromagnetic theory of light. But is this really a crucial result? A closer look at the theoretical implications of the experiments in air is necessary. Two points have to be outlined: a) are the electro-dynamical action-at-a-distance theories in direct contrast with a finite propagation in air? b) is Maxwell's theory really directly confirmed by the finite propagation in air?

a) Poincaré on this point in "La théorie de Maxwell et les oscillations Hertiennes" asserts that:

"Si en effet, il n'y a pas de courants de déplacement, si par conséquent il n'y a rien au point de vue électrique dans le diélectrique qui sépare le fil inducteur du fil induit, il faut bien admettre que l'effet se produit dans le fil induit au même moment

que la cause dans le fil inducteur; car dans l'intervalle, s'il y en avait un, la cause aurait cessée dans le fil inducteur, l'effet ne se serait pas encore produit dans le fil induit, et il n'y aurait rien dans le diélectrique qui est entre ces deux fils: il n'y aurait donc rien nulle part. La propagation instantanée de l'induction est donc une conséquence à laquelle l'ancienne théorie ne peut échapper."

But this, of course, is not a direct experimental result. It relies on the conceptual denial of the possibility of delayed action-at-a-distance. A denial is also implicitly expressed towards Helmholtz's intermediate position: dielectric plus action-at-a-distance. Poincaré seems to radicalise the split, and considers either action-at-a-distance with infinite speed or Maxwell's theory.

b) But what did Hertz prove exactly? In 1902 Poincaré asserts that Hertz did not directly prove Maxwell's fundamental idea, i.e. the action of a displacement current on a galvanometer. What Hertz directly proved was the finite propagation of induction. But:

"Seulement, supposer qu'il n'y a pas de courant de déplacement et que l'induction se propage avec la vitesse de la lumière; ou bien supposer que les courants de déplacement produisent des effets d'induction et que l'induction se propage instantanément, cela est la même chose."

Now the first two hypotheses are what Hertz proved, while the second two are what is specific to Maxwell's approach. Two remarks can be made. First, the second two hypotheses can be referred also to the delayed action-at-a-distance theories with dielectric, if polarisation currents are considered instead of displacement currents. Second, the first two hypotheses agree with delayed action-at-a-distance without dielectric. It is sufficient to compare with Poincaré's quotation above: if not displacement current, then what? The point is that the theory of desubstantialisation of the ether in the nineties reduces Hertz's support of Maxwell. Hertz's experiments in fact were considered crucial in the early nineties only for the acceptance of a substantial ether. Hertz's shift between 1887 and 1888 from the first to the third of the hypotheses of Maxwell's theory differing from Helmholtz, is itself based on the theoretical decision

to consider the third hypothesis (finite propagation in air) as inclusive of the first two (electromagnetic effects of polarisation and polarisation effects of induction). The denial of displacement currents that resulted in the developments of Maxwell's theory, reduced the differences between this theory and the opponents' one, as far as experimental grounds were concerned.

If Hertz's experiments were not crucial, we are left with the problem of explaining which were the reasons for the shift towards contiguous propagation. My point of view is that the reasons are strictly connected with Poynting's local conservation of energy and that this was recognized before Hertz's 1888 experiments.

But this is the beginning of another story.

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