

H.A.Lorentz, Theoretical Physics and the Unification of Electrodynamics:1870-1895

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Recent, mostly American, historical scholarship has dealt with the rise of theoretical physics in the second half of last century¹. But a definition of theoretical physics is still lacking². In my view this approach to physics research was born with Helmholtz's 1847 *Über die Erhaltung der Kraft* and was characterised by the conscious interplay of regulative principles and conceptual models with mathematical laws and experiments. That is, ever since 1847 physical laws had to be compared not only with experiments but also with specific general principles, and namely with the principle of energy conservation. In its turn the principle had various formulations and interpretations depending on the conceptual model adopted³.

H.A.Lorentz contributed to defining the task of theoretical physics. In fact, a methodological discussion of the role of conceptual models can be found in H.A.Lorentz's 1878 inaugural lecture⁴ to his course on Theoretical Physics. For him, the task of this discipline is to find simple basic principles from which all phenomena can be deduced, not to identify one specific conceptual model with "the truth", but to adopt a critical attitude in comparing different approaches: e.g. atoms and distant forces, contiguous action in a material ether, vortex ring atoms. Fifteen years later Lorentz was to show how to make good use of these methodological views in his successful attempt at unifying classical electrodynamics.

Lorentz was born in Arnheim in 1853. Already in his 1875 dissertation⁵ and in a paper of 1878⁶ he based a discussion of optical phenomena on a blend of electromagnetic theory of light and molecular physics. This blend was going to be a lasting element of his research program.

In 1877 Lorentz was appointed in Leiden to the first Dutch chair in Theoretical Physics. He held it for 35 years, till 1912. Then the physical cabinet of Teyler's Museum became his working place till his death in 1928.

In the 1880's Lorentz mostly worked on the molecular-kinetic theory of heat, but went back at the beginning of the nineties to electrodynamics.

In 1891 in an address⁷ to the Dutch Congress of Physics and Medicine he declared himself in favour of contiguous action, in 1892 he based his synthesis of electrodynamics⁸ on "mechanical principles", while in 1895 he assumed as axioms his own version⁹ of Maxwell's equations. Other relevant papers appeared in 1899¹⁰, 1904¹¹ and an exposition of Lorentz's Theory of Electrons was published in 1909¹². His series of eight volumes of lectures on Theoretical Physics was published in Dutch (1919-25), German (1927-31) and English (1931).

Lorentz's fame spread rapidly at the end of the century: in 1898 he accepted Boltzmann's invitation and addressed the Physics' section of the Düsseldorf's meeting of the German Society of Natural Scientists and Physicians, in 1900 he addressed the International Congress of Physics in Paris. In 1902 he received, together with P. Zeeman, the Nobel prize for his explanation of the 1896 Zeeman effect. He lectured in Berlin (1904), Paris (1905), New York (1906). From 1909 to 1921 he was the President of the Physics section of the Royal Netherlands Academy of Sciences and Letters, from 1911 to 1927 he regularly held the Presidency of the Solvay Conferences. From 1923 he became a member (and later the President) of the International Commission on Intellectual Cooperation of the League of Nations. At his death in 1928 Einstein, as representative of the Prussian Academy of Science, spoke at his graveside.

Lorentz is well known for many results, e.g.: the Lorentz-Lorentz formula on the relations between the density of a body and its index of refraction, the prediction of a discrete unit of electricity ("charged particle" in 1892, "ions" in 1895, "electrons" after 1899); the Lorentz force, the Lorentz contraction and the explanation of the Michelson-Morley experiments, the Lorentz transformations, the prediction of the variation of the mass with velocity, the explanation of the Fresnel drag coefficient (appreciated by Max Born as one of the "most beautiful examples of the might of mathematical analysis in the physical world"¹³).

I will confine myself here to only one aspect of Lorentz's scientific activity: the conceptual separation, in the early 1890's, of the electromagnetic field in the ether from matter. Einstein defined this achievement as an "act of intellectual liberation"¹⁴ and pointed out that:

"what in those times made it very difficult to grasp the essence of the electromagnetic theory was the following specific situation. The "intensities of the fields", electric and magnetic, and the "displacements" were both

treated as elementary quantities, and empty space as a special kind of dielectric body. *Matter*, not *space*, appeared as the carrier of the field."¹⁵

In fact Lorentz's innovation implied, as I am about to recall, a deep modification of all previous conceptual models in electrodynamics and at the same time provided a valuable synthesis.

From a contemporary point of view, the contiguous propagation of classical electromagnetic fields in empty space is accepted "naturally" and called "Maxwellian". But Maxwell contrasted the already existing and well-established continental theories of action at a distance with a theory of contiguous action that was based on the assumption of a fixed material ether pervading the whole space. Contiguous action was meant in a very material sense, in analogy with the mechanics of deformable bodies. Moreover the resulting delay in the propagation was not unique to Maxwell's approach. From the sixties on theories based on delayed action at a distance had been published. Lorentz's acceptance of a propagation in an empty ether implied thus a step backward towards the theories of delayed action at a distance in empty space. As Lorentz explicitly admitted:

"On voit donc que, dans la nouvelle forme que je vais lui donner, la théorie de Maxwell se rapproche des anciennes idées. On peut même, après avoir établi les formules assez simples qui régissent les mouvements des particules chargées, faire abstraction du raisonnement qui y a conduit et regarder ces formules comme exprimant une loi fondamentale comparable à celles de Weber et de Clausius. Cependant, ces équations conservent toujours l'empreinte des principes de Maxwell. Weber et Clausius regardaient les forces qui s'exercent entre deux atomes d'électricité comme déterminées par la position relative, les vitesses et les accélérations que présentent ces atomes au moment pour lequel on veut considérer leur action. Les formules, au contraire, auxquelles nous parviendrons expriment d'une part quels changements d'état sont provoqués dans l'éther par la présence et le mouvement de corpuscules électrisés; d'autre part, elles font connaître la force avec laquelle l'éther agit sur l'une quelconque de ces particules. Si cette force dépend du mouvement des autres particules, c'est que ce mouvement a modifié l'état de l'éther; aussi la valeur de la force, à un certain moment, n'est-elle pas déterminée par les vitesses et les accélérations que les petits corps possèdent à ce même instant; elle dérive plutôt des mouvements qui ont déjà eu lieu. En termes généraux, on peut dire que les phénomènes excités dans l'éther par le mouvement d'une particule électrisée se propagent avec une vitesse égale à celle de la lumière.

On revient donc à une idée que Gauss énonça déjà en 1845 et suivant laquelle les actions électrodynamiques demanderaient un certain temps pour se propager de la particule agissante à la particule qui en subit les effets."¹⁶

Thus Lorentz, despite other non-Maxwellian elements like the assumption of a unit of electric charge as the source of the interactions, claimed to be a Maxwellian.

Maxwell's *Treatise* was not a completely coherent book. It offered more than one view point. In what specific sense was Lorentz then a Maxwellian? In what sense did he differ from other Maxwellians like Hertz? What were the reasons for preferring contiguous action in empty space to delayed action at a distance? An answer to these problems is connected, in my view, not with experimental results but with the interplay between principles and models, a specific feature of the rapidly growing theoretical physics.

Relevant for the beginnings of Lorentz's research was Helmholtz's 1870 reformulation of electrodynamics. In the same year of his appointment to the Berlin chair in physics Helmholtz provided a framework, based on the action at a distance model, to compare some competing theories. His general expression for the energy of two current elements was¹⁷:

$$-\frac{1}{2}A^2\frac{ij}{r}[(1+K)\cos(Ds, D\sigma) + (1-K)\cos(r, Ds)\cos(r, D\sigma)]DsD\sigma$$

where A is a constant depending on the unit of current, r the distance between the elements of circuit Ds and Dσ traversed by the currents i and j. K too is a constant. Depending on the values of K different laws could be deduced.

For K=1: F. Neumann's 1845 potential¹⁸:

$$V=ij\iint_{s\sigma}\frac{(ds \times d\sigma)}{r}$$

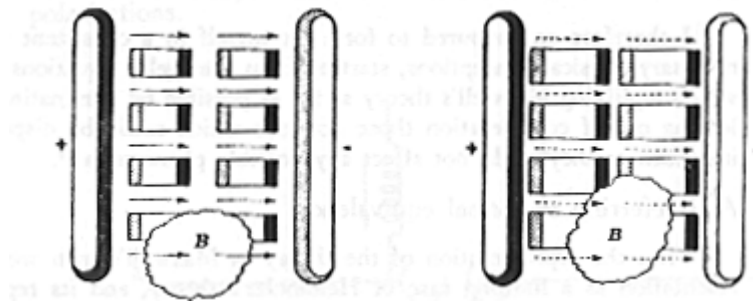
For K= -1: Weber's 1846 force law¹⁹:

$$F=\frac{e_1e_2}{r^2}\left[1-\frac{1}{c_w^2}\left(\frac{dr}{dt}\right)^2+\frac{2r}{c_w^2}\frac{d^2r}{dt^2}\right]$$

in electrostatic units, where velocity and acceleration are relative between the two particles at distance r, and the constant c_w expresses the ratio between the electromagnetic and electrostatic units of charge. In 1848 Weber published²⁰ a potential from which the force law could be deduced:

$$V=\frac{e_1e_2}{r}\left(\frac{1}{c_w^2}\frac{dr^2}{dt^2}-1\right)$$

For $K=0$ Maxwell's 1865 equations were derived, but not without some conceptual differences. These, hidden in Helmholtz's reformulation were beautifully schematized by Hertz in 1893²¹:



The figure shows two limiting cases of Helmholtz's theory: both refer to a conception of action at a distance starting from the charges on the plates that polarize the dielectric between the plates. In the first case energy is concentrated on the plates and direct action at a distance prevails; in the second case energy is in the dielectric and the charges on the plates produce a wave of polarisation. The second case is mathematically equivalent to Maxwell's equations. Helmholtz was very critical of Weber's approach, and based most of his criticisms on supposed inadequacies of Weber's law towards the principle of energy conservation. A footnote of Helmholtz's paper is particularly relevant: he suggested that Maxwell's analogy between the motions of electricity in dielectrics and the motion of light in the luminiferous ether might solve the difficulties of the elastic theory of reflection and refraction of light. This note was to be the explicit starting point for Lorentz's dissertation:

"Ce fut cette remarque de Helmholtz qui m'incita à chercher dans quelle mesure les phénomènes de réflexion et de réfraction de la lumière sont susceptibles de guider nos préférences plutôt du côté de la théorie de Maxwell, que vers la théorie ondulatoire admise jusqu'à présent."²²

Despite the conceptual differences, the competing theories were experimentally equivalent (only experiments on closed currents were available) and Helmholtz in 1879 urged Hertz, his best student ever, to find out the value of K experimenting on open currents.

Delayed action was not unique to Helmholtz's reformulation of Maxwell. Already Gauss in 1845 had pointed in this direction and laws of retarded potentials had been provided by Riemann in 1858, L.Lorenz in 1867, C.Neumann in 1868.

Riemann proposed for the first time a modified version of Poisson's law for the electrostatic potential U which is now accepted as correct:

$$\frac{d^2U}{dt^2} - \alpha \alpha \left(\frac{d^2U}{dx^2} + \frac{d^2U}{dy^2} + \frac{d^2U}{dz^2} \right) + \alpha \alpha 4\pi\rho = 0$$

where α is a velocity.

C. Neumann "succeeded brilliantly in showing how Weber's basic law was based on the assumption that the ordinary electrostatic potential spreads evenly to all sides with a certain speed, and that this distribution is the only cause for the electrostatic forces to appear dependent also on the speeds and accelerations of the effective elements of electricity"²³.

Betti too proposed a law of retarded potentials and Maxwell in 1873 acknowledged the formal equivalence of these competing approaches, and particularly of Lorenz, with his own²⁴.

In 1875 Clausius published a new contribution to the action at a distance approach based on a generalised potential

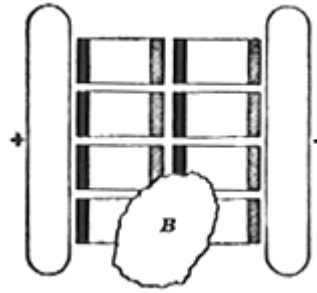
$$V = \frac{ee'(\mathbf{V} \times \mathbf{V}')}{r}$$

where \mathbf{v} , \mathbf{v}' are absolute velocities of the corpuscular charges e , e' .

Maxwell in 1873 published his set of field equations²⁵, which in great part had already appeared in 1865, utilising vector and scalar potentials. In modern notation:

A	Magnetic Induction	$\mathbf{B} = \text{curl } \mathbf{A}$
B	Electromotive Force	$\mathbf{E} = \nabla \wedge \mathbf{B} - \dot{\mathbf{A}} - \text{grad } \phi$
C	Mechanical Force	$\mathbf{F} = \mathbf{I} \wedge \mathbf{B} + e\mathbf{E} - m \text{grad } \Omega$
D	Magnetization	$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{J}$
E	Electric Currents	$4\pi\mathbf{l} = \text{curl } \mathbf{H}$
F	Current of Conduction	$\mathbf{I}' = \mathbf{C}\mathbf{E}$
G	Electric Displacement	$\mathbf{D} = (1/4\pi)\mathbf{K}\mathbf{E}$
H	True Currents	$\mathbf{I} = \mathbf{I}' + \dot{\mathbf{D}}$
J	Induced Magnetization	$\mathbf{B} = \mu\mathbf{H}$
K	Electric Volume Density	$\epsilon = \text{div } \mathbf{D}$
L	Magnetic Volume Density	$m = \text{div } \mathbf{J}$

In the *Treatise* he also outlined, not without contradictions, a more radical contiguous action model, with priority given to the polarisations and not to the charges. With this model, action at a distance and electricity as an incompressible fluid are abandoned. In Hertz's scheme :



In Hertz's views:

"Maxwell's own representation ... frequently wavers between the conceptions which Maxwell found in existence, and those at which he arrived. Maxwell starts with the assumption of direct actions-at-a-distance; he investigates the laws according to which hypothetical polarisations of the dielectric ether vary under the influence of such distance-forces; and he ends by asserting that these polarisations do really vary thus, but without being actually caused to do so by distance-forces."²⁶

The *Treatise* clearly shows the evolution of Maxwell's ideas: moving from the already established results of action at a distance theories, based on the primary role of the charges, Maxwell introduced the electric displacement in a polarised material ether, caused by action-at-a-distance forces, and finally introduced contiguous action as an autonomous entity. But Maxwell's links with the mechanical world view remained very strong: in 1873 despite abandoning the search for a mechanical model of the ether he attempted a derivation of the field equations from "mechanical principles", that is, a Lagrangian derivation. He adopted Helmholtz's 1847 mechanical principle of energy conservation, but he "localised" the energy in the medium. He identified potential with electrostatic energy and kinetic with magnetostatic. He aimed at a new energy conservation law, in the form of a continuity equation, but did not reach it. Maxwell interpreted the concept of electromagnetic field in a very material sense, as a part of the material ether.

In the seventies, Helmholtz, now a leader of European physics, was in a difficult position: his original 1847 Newtonian force law approach had been rejected by all competitors. Moreover the sharp distinction between kinetic and potential energy was questioned by the generalised potentials and the retarded potentials. Among the competitors Maxwell provided, despite the finite velocity of the interactions, a "localisation" of energy of great appeal for a "mechanical" world view, and so Helmholtz was more

inclined towards this limiting case of his own reformulation. But this limiting case should not be confused with Maxwell's "radical" conceptual model, where primacy is given to polarisations and not to charges.

Lorentz started his researches from Helmholtz's suggestions and understood Maxwell through Helmholtz's reformulation: charges producing at a distance waves of polarisations. But from the beginning, Lorentz, who was explicit on the relevance of the interplay between different conceptual models, hinted at a synthesis of corpuscular electricity and contiguous action. More importantly, from the 70's he started conceiving of the possibility of a contiguous propagation in empty space, a rather non mechanical concept, difficult to distinguish from delayed action at a distance.

Hertz too started from Helmholtz's reformulation ("I have rather been guided by Helmholtz's work."²⁷), but when he finally managed in 1887-90 to conceive and perform the experiments whose necessity Helmholtz had stressed, he shifted towards the Maxwell's more radical contiguous action point of view of Maxwell:

"...an attempt has been made, in the two theoretical papers here printed, to exhibit Maxwell's theory, i.e. Maxwell's system of equations, from this fourth stand-point."²⁸

In the two theoretical papers of 1890 he developed his views, assuming an ether dragged with the moving bodies and refusing a mechanical derivation of the field equations. Nevertheless in 1893 he honestly asserted to have only proved experimentally the time delay of the interactions and not one or the other theory:

"Casting now a glance backwards we see that by the experiments above sketched the propagation in time of a supposed action-at-a-distance is for the first time proved. This fact forms the philosophic result of the experiments; and, indeed, in a certain sense the most important result....

What we here indicate as having been accomplished by the experiments is accomplished independently of the correctness of particular theories."²⁹

Hertz's experiments were not crucial. This can be shown by Poincaré's difficulties from 1890 to 1902 in interpreting them: the delay in the propagation could be explained as due to an instantaneous action at a distance plus a time for the polarisation or to a contiguous interaction³⁰.

Lorentz in 1892 took a step different from that of Hertz: starting from Helmholtz's limiting case of Maxwell's theory, he attempted to improve Maxwell's mechanical derivation, assumed elementary electrical charges as sources of a delayed action in empty space (the great conceptual novelty) and of the polarisation of ponderable bodies. He split, in fact, the concept of ether from that of matter and assumed a fixed ether as an absolute reference frame. We saw that he himself in 1892 asserted to have in a way provided a scheme equivalent to Clausius' and Weber's and to have gone back to Gauss's old idea of delayed action at a distance. In 1895 Lorentz abandoned the mechanical derivation of his equations, which maintained many non-Maxwellian characteristics, like Lorentz's force which depends on the velocities of charges. His *Theory of Electrons* was a real synthesis between the two great traditions:

$$\begin{aligned} \operatorname{div} \mathbf{d} &= \rho; \operatorname{div} \mathbf{h} = 0; \\ \operatorname{rot} \mathbf{h} &= \frac{1}{c} \mathbf{c} = \frac{1}{c} (\dot{\mathbf{d}} + \rho \mathbf{v}); \operatorname{rot} \mathbf{d} = -\frac{1}{c} \dot{\mathbf{h}}; \\ \mathbf{f} &= \mathbf{d} + \frac{1}{c} [\mathbf{v} \cdot \mathbf{h}] \end{aligned}$$

Why did Lorentz claim to be a Maxwellian? The conventional answer is that he assumed the finite propagation of the interactions, experimentally proved by Hertz. But time delay was also assumed by the delayed action at a distance theories (like Lorentz's) and Hertz's experiments could not give direct proof of one kind of propagation versus the other. The formal equivalence between delayed action at a distance and contiguous action already noted by Maxwell, Lorentz himself and Hertz, was later stressed by Schwarzschild in 1903, Whittaker in 1910, and more recently by Feynman, who reasserted a delayed action at a distance force law close to Weber's³¹.

But of course, Lorentz had, if not an experimental, a good theoretical reason to prefer contiguous action: the specific version of the principle of energy conservation connected to the contiguous action theory. As he recalled in 1923:

"It was not always easy to grasp Maxwell's ideas, and one feels a want of unity in his book, due to the fact that it faithfully reproduces his gradual transition from old to new ideas...."

Maxwell's followers, of whom there were many, in this country and elsewhere, have perfected the theory in its form and extended it by the introduction of new ideas. Think, for instance, of Poynting's beautiful and important theorem on the flow of energy, determined at every point by the electric and magnetic force existing in the field, a theorem that has produced more clearness perhaps than any other and which is now so essential that we can hardly recall the state in which physics was when we did not know of it. Yet, notwithstanding all innovations of this kind, we always speak, and with full justice, of 'Maxwell's Theory'. We continue to do so now that we have been led to introduce electric charges supposed to exist in the interior of molecules and atoms, by which we have come to the theory of electrons....

.....what I want to point out is this, that we could never have gone so far if we had contented ourselves with the actions at a distance, if we had not fixed our attention on the intervening medium, localising the energy in it and considering it as the seat of momenta and stresses which manifest themselves in the observed motions of bodies. All these modern ideas have their origin in Maxwell's work."³²

Poynting had proposed his "theoretical" development of Maxwell's theory in 1884-5³³. He was aware of the need to overcome a mechanical concept of "displacement" and thus to come closer to contiguous propagation in empty space. These non mechanical aspects were criticised by J.J.Thomson in 1885. But Planck,in 1887, fully realised the great simplification based on the contiguous propagation of energy ("near" action)³⁴.

The fertility of this simplification was recognised by Lorentz in 1891, when he declared his conversion to contiguous action:

"Thus we are led (in Weber's and Clausius' theories) to assume an energy which depends on the velocity of particles, and nevertheless is not kinetic energy in the ordinary sense of the word. This is a major difficulty: although it is simpler to denote energy dependent on velocity as kinetic energy, here it is not the case. In Maxwell's conception it is considered as such, and in this I find the first reason to give his theory precedence."³⁵

Lorentz thus managed to overcome traditional mechanical ideas modifying conceptual models on the grounds of their relations with

regulative principles, in the style of the "new" theoretical physics. He opened the way for the short but glorious synthesis known as the "electromagnetic view of nature" which, also through the efforts of Wiechert, Wien and Abraham, flourished before the success of the Theory of Relativity and Quantum Theory between 1900 and 1910³⁶.

¹McCormmach, R. "Editor's Foreword" In *Historical Studies in Physical Sciences* 3 (1971) ix-xxiv; Olesko, K. "The Emergence of Theoretical Physics in Germany: Franz Neumann and the Königsberg School of Physics, 1830-1890." Dissertation. Cornell University, 1980; C.Jungnickel, R.McCormmach, *Intellectual Mastery of Nature*, 2 vols, Chicago: Chicago U.P., 1986; Wolff,S."Clausius' way to the kinetic theory-the beginnings of theoretical physics in Germany." Forthcoming

²C.Jungnickel and R.McCormmach's title is a quotation from Helmholtz's autobiography, but a precise definition of the term "theoretical physics" widely used in the subtitles ("Theoretical Physics from Ohm to Einstein; vol 1: The Torch of Mathematics 1800-70; vol 2: "The Now Mighty Theoretical Physics 1870-1925") is lacking. References are made to Boltzmann (1895): "Even the formulation of this concept is not entirely without difficulty," Vol.1 p.XV; to Wien (1905): vol.2 p.XV. Wolff too, facing the same problem, relies on Boltzmann: n. 89.

³Bevilacqua, F. "Conservation and Innovation : Helmholtz's Struggle with Energy Problems (1845-1894) and the Birth of Theoretical Physics." In *H.v.Helmholtz* D.Cahan ed. California University Press. Forthcoming.

⁴Lorentz, H.A. 1878 "Molecular Theories in Physics." In *Collected Papers* Vol.9. Pp.26-49. Most of Lorentz's papers have been republished and some translated in: Lorentz, H.A. *Collected Papers*. Eds. P.Zeeman and A.Fokker. 9 vols. The Hague 1935-39. A detailed scientific biography of Lorentz is still lacking, but see: McCormmach,R. "Lorentz, Hendrik Antoon" in *Dictionary of Scientific Biography*. Vol.8 1973. Pp.487-500; it includes a bibliography of secondary sources.

⁵Lorentz, H.A. 1875 "Sur la théorie de la réflexion et de la réfraction de la lumière." 1875 In *CP* Vol.I Pp.193-383.

⁶Lorentz, H.A. 1878 "Concerning the Relation between the Velocity of Propagation of Light and the Density and Composition of Media." In *CP* Vol.2. Pp.1-119.

⁷Lorentz, H.A. 1891 "Electriciteit en Eether." In *CP* Vol.9. Pp.89-101.

⁸Lorentz, H.A. 1892 "La théorie électromagnétique de Maxwell et son application aux corps mouvants." In *CP* Vol.2. Pp.164-343.

⁹Lorentz, H.A. 1895 "Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern." In *CP* Vol.5. Pp.1-138.

¹⁰Lorentz, H.A. 1899 "Théorie simplifiée des phénomènes électriques et optiques dans des corps en mouvement." In *CP* Vol.5. Pp.139-155.

¹¹Lorentz, H.A. 1904 "Electromagnetic Phenomena in a System Moving with any Velocity Smaller than That of Light." In A.Einstein et al.*The Principle of Relativity*, tr.W.Perrett and G.B.Jeffrey. New York: Dover. Pp.11-34.

Lorentz, H.A. 1904 "Weiterbildung der Maxwellschen Theorie. Elektronentheorie." In *Encyklopädie der mathematischen Wissenschaften*. Leipzig. Vol.5. Nr.14. Pp.145-280.

¹²Lorentz, H.A. 1909 *The Theory of Electrons* 2nd ed 1915. Rep. New York: Dover, 1952.

¹³McCormmach, see n.1 p.495.

¹⁴McCormmach, see n.1 p.498.

¹⁵Einstein, A. "Autobiographical Notes". In *A.Einstein: Philosopher-Scientist*. P.A.Schilpp ed. Evanston 1949 (my retranslation from Italian).

¹⁶Lorentz, H.A. 1892 n.8 p.229.

¹⁷Helmholtz, H. "Ueber die Bewegungsgleichungen der Elektrizität für ruhende leitende Körper." In *Borchardt's Journ* 72 (1870): 57-129; repr. in *WA* 1 pp.545-628. Pp. 549 and 567.

¹⁸Neumann,F. *Berlin Abhandlungen* (1845) P.1.

¹⁹Weber, W. "Elektrodynamische Maassbestimmungen über ein allgemeines Grundgesetz der Elektrischen Wirkung." In *Werke* 6 vols. Berlin, 1892-4. Vol.3, pp.25-214.

²⁰Weber, W. "Elektrodynamische Maassbestimmungen" In *Pogg Ann* 73 (1848): 193. P.229. Tr. in *Taylor's Scientific Memoirs* 5 (1852): 489-529.

²¹Hertz, H. *Untersuchungen über die Ausbreitung der elektrischen Kraft*. Leipzig, 1892. Tr. by D.E.Jones *Electric Waves*. London: Mac Millan, 1892. Repr. New York: Dover, 1962. Pp.24-5.

²²Lorentz, H.A. 1875 n.5 P.194.

²³Planck,M.*Das Prinzip der Erhaltung der Energie* . Leipzig: Teubner, 1887. 2nd ed 1908. P.241.

²⁴Maxwell, J. C. *A Treatise on Electricity and Magnetism*. 2 Vols. 1873; Thomson J.J. ed of 3rd edition 1891; repr New York: Dover,1954. Vol.2 Pp. 450 and 490-3.

²⁵Maxwell,J.C. Ibidem Vol.2 Pp.247-59. Table reproduced from Everitt, C.W.F. "Maxwell,J.C." In *DSB* Vol.9 (1974) Pp.198-230. P.212.

²⁶Hertz,H. n.21 P.195.

²⁷Ibid p.20.

²⁸Ibidp.27.

²⁹Ibid p.19.

³⁰Bevilacqua,F. "H.Hertz's Experiments and the Shift Towards Contiguous Propagation in the Early Nineties." In *Rivista di Storia della Scienza* 1 (1984): 239-56.

³¹Feynman, Richard. *The Feynman's Lectures on Physics*. 3 vols. Addison Wesley, 1963. V.2 Chapt.21.

³²Lorentz,H.A. 1923 "Clerk Maxwell's Electromagnetic Theory." In *C P* Vol.8 Pp.356 and 360.

³³Poynting,J.H. (1884) "On the Transfer of Energy in the Electromagnetic Field." In *Philosophical Transaction of the Royal Society* (1884): 343-361; "On the connexion between Electric Current and the Electric and Magnetic Inductions in the surrounding field." In *Phil. Trans.* (1885) 277-306.

³⁴Bevilacqua, F. *The Principle of Conservation of Energy and the History of Classical Electromagnetic Theory*. Pavia: La Goliardica Pavese, 1983.

³⁵Lorentz,H.A. 1891 n.7. Quoted in Hirosige,T."Origins of Lorentz Theory of Electrons and the Concept of Electromagnetic Field." *H S P S* Vol.1 Pp.151-209. P.183.

³⁶McCormmach,R."H.A.Lorentz and the Electromagnetic View of Nature." In *Isis* 61 (1970):459-97.