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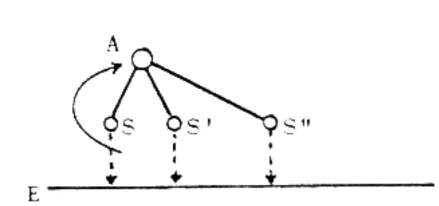
Textbook’s physics versus history of physics: the case of Classical Electromagnetic Theory

Part I

A great cultural inheritance for the physics students, lecturers and historians is the set of textbooks on theoretical physics written by great physicists or Nobel prize winners as Planck, Sommerfeld, Pauli, Landau and Lifchitz, and Feynman. But the textbooks on Classical Electromagnetic Theory (C.E.T.) offer puzzling features to the ones who identify textbook science with normal science, in the Kuhnian sense. The five C.E.T. textbooks of the mentioned authors in fact assess C.E.T. results in a number of conceptually different ways and this leads to a certain ambiguity as to the theoretical roots of the subject. The aim of the first part of this paper is to give examples of these different approaches and the aim of the second part is to relate the different approaches to the historical debate on C.E.T. of the second half of the 19th century. The result will be a flow chart that gives at the same time an historical explanation of the different approaches of the textbooks and a physical assessment of the historical contributions of the different schools involved in the "building" of C.E.T. To frame 20th century textbooks and 19th century papers in a coherent way a partially new methodology is needed. It can be easily deduced by some methodological assertions of A. Einstein. These are presented in a letter of Einstein’s published recently: the letter of May 7th, 1952 to Maurice Solovine (11). In that letter a scientific system is conceived as a three-level structure, by means of the following diagram:



A System of axioms

S Resultant propositions

E Variety of immediate sense-experiences

Fig. 1

The relations between the axioms (level A) and the propositions comparable with experience (level S) are of a logical nature and for physics are mathematical and formal; but for Einstein the connections between S and E (verification and possible falsification of resultant propositions) and between E and A (construction of axioms on the ground of experience) are not of a logical nature. He firmly rejected the classical conception of the positivistic model, according to which propositions and, indirectly, even basic axioms, are constructed, demonstrated and verified on the ground of experiments. According to Einstein, scientific procedure develops in a completely different way:

> "The simplest conception (model) one might make oneself of the origin of a natural science is that according to the inductive method. Separate facts are so chosen and grouped that the lawful connection between them asserts itself clearly ... But a quick look at the actual development teaches us that the great steps forward in scientific knowledge originated only to a small degree in this manner. For if the researcher went about his work without any preconceived opinion, how should he be able at all to select out those facts from the immense abundance of the most complex experience, and just those which are simple enough to permit lawful connections to become evident?" (12)

In the most important scientific papers it is always possible to find the employment of these unverifiable, unfalsifiable but not arbitrary conceptions of hypotheses. They are as necessary to research as the empirical and analytical aspects.

The Einsteinian scheme greatly reduces the difference between a static and a dynamic presentation of science, i.e. between normal and extraordinary science. The role of the "preconceived opinion" needed to relate sense experiences to axioms and resultant propositions to sense experiences introduces an historical, not strictly logical element in the scheme. The conceptual models and the regulative principles are in fact deeply rooted in the development of science itself.

In the literature, Einstein’s methodology has been deeply analysed (13), and from this scheme a three-components view of science has been outlined (14). Other analogous threecomponents schemes have been derived from the philosophical tradition (15). In this analysis I shall utilise a four-component scheme: the regulative principles, the conceptual models, the mathematical formalism and the experimental results.

"Regulative principle" is here used with reference to the heuristic and justificative role that statements with metaphysical value play in scientific theories. The heuristic role promotes the discovery of new specific laws, the justificative role gives the possibility of reducing already known laws to theories. The interactions between the four components can be analysed both in a dynamical sense (some of these components change while others remain constant in a given time interval) and in a static sense (the four components together represent a scientific theory at a given instant).

This approach includes metaphysical aspects as a necessary part of scientific development but does not analyse the philosophical, psychological, social, economic, technological (and so on) origins of the four components. The focus is directed on the mutual interplay of the components to the so-called "hard core" of science. An analysis of the textbooks of some modern Nobel prize winners will provide a variety of contemporary interpretations of each component, useful for a broader approach to a historical reconstruction (16). But the attempt to analyse the interplay of the interpretations of each component with the interpretations of the other three would simply mean a new attempt at writing a total history. Thus attention has been focused only on the relations of the component of regulative principles (and mainly of the Principle of Conservation of Energy (PCE)) with the component of conceptual models.

I.2) Contemporary Textbooks

The four-component image of science helps us to find a precise answer to the question: what is CET today? As far as modern CET is concerned, in fact, the interplay between the four components is evident in every exposition of the theory, even if it is different in each case. The conceptual components refer to the two main competing models: the contiguous versus the action-at-a-distance, the field versus the particle, the continuous versus the discrete conception of nature. I will show that some leading physicists (Sommerfeld, Landau) prefer the first model, one underlines the problematic synthesis of the two (Pauli), and one prefers the second model (Feynman).

The regulative component in CET is mainly connected with the Principle of Conservation of Energy (PCE) and the Principle of Least Action (PLA). Sometimes PLA is assumed as the basic principle (Landau) and sometimes this role is reserved for PCE (Planck). In relation to the mathematical component, almost all the authors underline the great role of the Mathematical Potential Theory (MPT) in CET. Much attention is given to the wide range of applications of analogous equations (Sommerfeld, Feynman), as well as to the mathematical equivalence established through MPT between the two different basic sets of equations referring to contiguous action and action-at-a-distance (Feynman).

As far as the experimental component is taken into account, two aspects must be noted: a naïve inductive use of experiment-theory relations and a more sophisticated use of ’experimental’ results in the determination of the specific form of the regulative principle utilised.

A) Mathematical Component.

The Mathematical Potential Theory (M.P.T.) has great importance in the development of CET and a wide application in both classical models in physical science: particle and continuous-media physics. In the development of MPT both of these old, basic conceptions are utilised. On one hand, continuous bodies are sometimes considered ultimately as aggregates of particles, to which Newton’s law can be generalised. On the other hand, the forces at all points of space are considered as determined by a continuous distribution of potential surfaces, rather than fixing attention on the forces at isolated points. The motivation for the widespread use of MPT seems two-fold. First, it allows the description of vectorial (and sometimes tensorial) quantities in terms of scalar or simpler quantities. Thus, a conservative force F can be expressed as the gradient of a scalar. This is a basic simplification of description and it gives a much more intuitive picture of the field in terms of equipotential surfaces. Second, it leads naturally from the concept of action-at-adistance (the force upon a given body being the sum of many contributions from all other bodies) to the concept of a field (the force is eventually determined not by a sum or an integral, but by a partial differential equation). This partial differential equation can be formulated only in terms of the potential.

This double application of MPT is present also in CET, where it is applied both to actionat-a-distance and to contiguous action approaches. This situation causes some difficulties for the physical interpretations of the quantities involved. The formal role of MPT, as an intermediate formulation of different conceptual models, was called by Einstein and Infeld "the field as representation" (17) and by Max Born "the pseudocontiguous action" (if time is not involved) (18). The wide range of MPT's applications has sometimes raised questions. Is there a pre-established harmony between mathematical physics and the physical world? Is there an underlying unity in the universe, reflected in the generality of potential equations? Sommerfeld's and Feynman's textbooks deal at length with these problems.

Arnold Sommerfeld in the foreword to his textbook on partial differential equations in physics asserts:

> "The oftmentioned 'prestabilized harmony' between what is mathematically interesting and what is physically important is met at each step and lends an aesthetic - I should like to say metaphysical - attraction to our subject." (19)

It is extremely interesting to see the widespread use of one well-known group of equations: the Laplace and Poisson second degree partial differential equations, the socalled potential equations:

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

and

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = -4\pi\rho$$

where U is the potential and p a density.

These equations are well known in several branches of physics referring both to particles and continuous media. They are used in the theory of gravitation, in electrostatics and magnetostatics, in the hydrodynamics of incompressible and irrotational fluids (where U stands for the velocity potential). The two dimensional potential equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

is the basis of Riemannian function theory, which Sommerfeld characterises as the ’field theory’ of the analytic functions. The wave equation

$$\Delta U = \frac{1}{c^2} + \frac{\partial^2 U}{\partial t^2}$$

is fundamental in acoustics (c=velocity of sound) and in the electrodynamics of variable field (c=velocity of light) and in optics. In the special theory of relativity, it becomes the four-dimensional potential equation

$$\Box U = \sum_{K=1}^{4} \frac{\partial^2 U}{\partial x_k^2} = 0$$

Other applications are to equilibrium states and oscillating processes (elasticity theory: oscillating membrane and string, transverse vibrations of a thin disc, oscillating elastic rod, etc.), and also to equalisation processes: heat conduction (equalisation of energy differences), diffusion (equalisation of differences of material densities), fluid friction (equalisation of impulse differences) and pure electric conduction (equalisation of differences of potential). Schroedinger’s equation of wave mechanics belongs formally to the same scheme, in particular in the force-free case:

$$\Delta U = \frac{2m}{ih} \frac{\partial U}{\partial t}$$

where m=mass of the particle, h=Planck’s constant divided by 2π.

This widespread application of the potential equation without doubt justifies Sommerfeld’s central claim for a "pre-established harmony". But it requires a deeper analysis than the assertion that it stems "from the invariance under rotation and translation which must be demanded for the case of isotropic and homogeneous media", and that the use of "partial differential equations is due to the field-action approach, which is the basis of present-day physics, according to which only neighbouring elements of space can influence each other". (20)

R. Feynman points out that we are dealing with a continuous distribution in an isotropic and homogeneous space:

> "Is it possible that this is the clue? That the thing which is in common to all the phenomena is the space, the framework into which the physics is put? As long as things are reasonably smooth in space, then the important things that will be involved will be rates of change of quantities with position in space. That is why we always get an equation with a gradient or a divergence; because the laws of physics are independent of direction, they must be expressible in vector form ... what is common to all our problems is that they involve space and that we have imitated what is actually a complicated phenomenon by a simple differential equation." (21)

Feynman’s remarks, written thirty years after Sommerfeld’s, present a more abstract approach: they do not refer to isotropic and homogeneous media but to space as framework of all phenomena, continuous and discontinuous , and the so-called field approach is referred to as the (in quantum terms) probably approximate condition of continuous space. Any similarity of basic substances is explicitly denied:

> "The ’underlying unity’ might mean that everything is made out of the same stuff, and therefore obeys the same equations. That sounds like a good explanation, but let us think. The electrostatic potential, the diffusion of neutrons, heat flow - are we really dealing with the same stuff? Can we really imagine that the electrostatic potential is physically identical to the temperature, or to the density of particles? Certainly it is not exactly the same as the thermal energy of particles. The displacement of a membrane is certainly not like a temperature. Why, then, is there ’an underlying unity’?" (22)

The rejection of "pre-established harmony" and of the "underlying unity" shifts attention to the scientist’s use of MPT and to the possibilities it offers of a formal translation of results belonging to different conceptual frameworks. In CET this translation is utilised quite often, but usually without clarity about the simultaneous conceptual shift in meaning. An important exception is offered by Feynman’s discussion of the modern action-at-a-distance formulation and of its formal equivalence with Maxwell’s equations.

Feynman in 1949 derived a set of two equations that represent the electric and magnetic field produced by a single individual charge. "So if we can find the $\underline{E}$ and $\underline{B}$ produced by a single charge, we need only add all the effects from all the charges in the universe to get the total $\underline{E}$ and $\underline{B}$." (23)

That is, the law is interpreted according to the action-at-a-distance model, the fields $\underline{E}$ and $\underline{B}$ being only the abbreviations needed in the expression of Lorentz’s force on a single particle: $F=q(\underline{E}+\underline{v}\times\underline{B})$. The full expression is the direct charge-to-charge interaction:

$$\underline{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\underline{e}_{r'}}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\underline{e}_{r'}}{r'^2}\right) + \frac{1}{c^2}\frac{d^2\underline{e}_{r'}}{dt^2}\right]$$

$$c\underline{B} = \underline{e}_{r'} x \underline{E} \qquad (1)$$

where q is the charge that is producing the field, $\underline{e}_r$' is the unit vector in the direction towards the point 1 where E is measured, r is the distance from 2 to 1, r' is the distance from 2' to 1 when the information now arriving at 1 left q: the fields at (1) at the time t depend on the position (2') occupied by the charge q at the time t'=(t-r'/c).

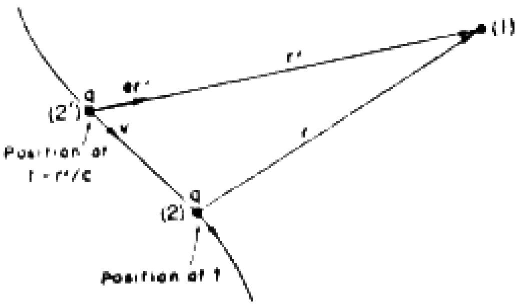


Fig. 2

(Source: Feynman (1963))

The laws are completely equivalent to Maxwell's equations: "This law for the fields of an individual charge is complete and accurate, so far as we know (except for quantum mechanics)" (24). In spite of retaining the conceptual framework of action-at-a-distance, it diverges from the Newtonian model of central forces: the interaction between charges depends not only on positions but also on velocities and accelerations. Moreover, the distances are measured at the retarded time, that is, a finite speed of propagation of interactions is assumed. The equivalence with Maxwell's equations depends on the fact that both Maxwell's and Feynman's equations can be solved through the retarded potentials:

$$\phi(1, t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(2,t')}{r12}\, dV_2$$

and

$$\underline{A}(1, t) = \frac{1}{4\pi\varepsilon_0 c^2} \int \frac{\underline{J}(2,t')}{r12}\, dV_2$$

with

$$t' = t - r12/c$$

which are solutions of d’Alembert’s equations for ϕ and A. All this draws attention to an important feature of CET: the mathematical equivalence, through the Mathematical Potential Theory (MPT) in the retarded potentials version, between the different models of contiguous action and action-at-a-distance.

B) Conceptual Models component

Standard textbooks in CET have common elements which belong to two different conceptual models: the particles acting at a distance and the field acting contiguously. Owing to the mathematical possibility of translation of one model into the opposite, as analysed above, some confusion arises when the interpretations are not properly clarified. The general need to have both models is in fact unquestionable, because it is impossible to reduce all the results under one single model, but the confused interpretations of single formal results can be avoided.

The need for the two models, sometimes in conflict, is expressed by Pauli:

> "it is, however, by no means true that field physics has triumphed over corpuscular physics, as can be seen from the fact that electricity is atomistic in nature ... The existence of an elementary charge has, until now, in no way been made plausible. It is still an open problem in theoretical physics. The electron itself is a stranger in the MaxwellLorentz theory as well as in the present-day quantum theory" (25)

and

> "The field-particle description presents a conceptual problem: although a field can be described mathematically without the need for any test charges, it cannot be measured without them. On the other hand, the test charge itself gives rise to a field. However, it is impossible to measure an external field with a test charge and, at the same time, to determine the field due to this charge. A certain duality exists. Consequently, electrodynamics is of great significance for physical epistemology." (26)

Coulomb’s law is an example of confusion arising from the mixture, sometimes in different chapters of the same book, of different interpretations: there is in fact a conceptual difference between Coulomb’s Newtonian law of electrostatics and Maxwell’s contiguous action equations of electromagnetism. Some textbooks present Coulomb’s law as an experimental result, others deduce it from Maxwell’s equations div $\underline{E}=4\pi\rho$ and curl $\underline{E}=0$. Of course Maxwell’s equations are still valid in the electrostatic approximation, but the conceptual meaning of Coulomb’s law appears to be different if presented as an autonomous result, "experimentally" discovered in 1785 and still valid at a high degree of approximation or as a deduction from Maxwell’s equation valid within specific limits. To clarify this point it is useful to speak in terms of potential rather than in terms of fields. The potential equation connected with Maxwell’s two electrostatic equations div $\underline{E}=4\pi\rho$ and curl $\underline{E}=0$ is a Poisson equation

$$\Delta\phi = -4\pi\rho$$

Instead the potential (scalar) equation connected with Maxwell's electromagnetic equations is the d'Alembert equation:

$$\Delta\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -4\pi\rho$$

Given the boundary conditions in the first case (potential of a Coulombian field) the potential is:

$$\phi = \frac{e}{R} \qquad (1)$$

In the second case (scalar potential of a Maxwellian field) the potential is:

$$\phi = \frac{e}{\left(R - \frac{\underline{V}\cdot\underline{R}}{c}\right)} \qquad (2)$$

Of course (2) can be reduced to (1) in the static case. But this does not imply action-at-adistance in Coulombian fields derived from Maxwell's equations. In fact there is a Coulombian field approximation also in a non-stationary case: for R→O the potential itself goes to infinity and in consequence its spatial derivatives increase more than the time derivatives. Thus for R→O the time derivatives in d'Alembert's equation can be dropped and the equation itself reduces to Poisson's. (27)

What is extremely interesting is Feynman's remark about Coulomb's law. In fact, the first term of his equation, $\underline{E} = q\underline{e}_{r'}/4\pi\varepsilon_0 r'^2$ represents the static approximation and it should be equivalent to Coulomb's law. But "Coulomb's law is wrong":

> "The discoveries of the 19th century showed that influences cannot travel faster than a certain fundamental speed c, which we now call the speed of light. It is not correct that the first term is Coulomb's law, not only because it is not possible to know where the charge is now and at what distance it is now, but also because the only thing that can affect the field at a given place and time is the behaviour of the charges in the past. How far in the past? The time delay, or retarded time, so-called, is the time it takes, at speed c, to get from the charge to the field point P. The delay is r'/c." (28)

This point is relevant for my purposes: the analysis of Coulomb's law from a retarded action-at-a-distance point of view as formally equivalent to Maxwell's equations, casts a new light on what a principle of correspondence means (that is, Coulomb's law as approximation from Maxwell's equations), and what constitutes a meaning shift (that is, the standard interpretation of Coulomb's law cannot be derived from Maxwell's equations, for it implies action-at-a-distance instead of contiguous action). It is remarkable that an explicit statement of the fact that Coulomb’s law is conceptually wrong comes from a textbook that stresses the relevance of action-at-a-distance, even if delayed. In fact in Feynman’s words, the first two terms of (1) together give the instantaneous Coulomb field:

> "The second term says that there is a ’correction’ to the retarded Coulomb field which is the rate of change of the retarded Coulomb field multiplied by r'/c, the retardation delay. In a way of speaking, this term tends to compensate for the retardation in the first term. The first two terms correspond to computing the ’retarded Coulomb field’ and then extrapolating it towards the future by the amount r'/c, that is, right up to the time t! The extrapolation is linear, as if we were to assume that the ’retarded Coulomb field’ would continue to change at the rate computed for the charge at the point (2'). If the field is changing slowly, the effect of the retardation is almost completely removed by the correction term, and the two terms together give us an electric field that is the ’instantaneous Coulomb field’ - that is, the Coulomb field of the charge at the point (2) - to a very good approximation." (29)

The third term of (1) accounts for radiation and is not relevant to this discussion of Coulomb’s law. What has been illustrated is the confusion in textbooks between the principle of correspondence of quantitative results and the principle of translation of meanings.

Further confusion arises from an unclear definition of the basic quantities, as well as from an unclear distinction between mathematical and physical quantities. Feynman for instance outlines a non-standard approach. After the breakdown of the Einstenian unifying programme and the development of quantum physics, the physical model of the continuous field based on the space-time geometrical model becomes, in his view, much less important. Not only does the Newtonian concept of force becomes less important, but so also does the Einsteinian continuous field:

> "... the vector potential A (together with the scalar potential d that goes with it) appears to give the most direct description of physics. This becomes more and more apparent the more deeply we go into the quantum theory. In the general theory of quantum electrodynamics, one takes the vector and scalar potentials as the fundamental quantities in a set of equations that replace the Maxwell equations: E and B are slowly disappearing from the modern expression of physical laws: they are being replaced by A and ϕ". (30)

Sommerfeld’s point of view is different, relying on Hertz’s analysis of Maxwell’s theory and on his reformulation of it. The reformulation involved the elimination of potentials from the basic equations and Sommerfeld refers to it as a "purification":

> "Here Maxwell’s equations, purified by Heaviside and Hertz were made the axioms and the starting points of the theory. The totality of electromagnetic phenomena is derived from them systematically by deduction. Coulomb’s law, which formerly provided the basis, now appears as a necessary consequence of the all-inclusive theory..."

and

> "The simplified form of the Maxwell equations, later rediscovered by Heaviside and Hertz, is to be found already in Part III of his paper for the Royal Society of 1864." (31)

Thus the potentials are considered second level, spurious elements that have to be eliminated from the equations in order to "purify" them.

Apart from the evaluation of the role of fields and potentials (that is E and B versus A and ϕ) there is another problem about these quantities: could the fields E and B be considered as physical quantities, and A and ϕ as mathematical ones?

Actually the shift from Poisson's to d'Alembert's equation does not necessarily imply the shift from action-at-a-distance to contiguous action, and thus, given the theories of delayed action-at-a-distance, the finite speed of propagation of interaction does not imply the field concept and its physical 'reality'. From this point of view, the differences between physical models of fields and mathematical potentials tend to disappear, since both fields and potentials can be considered as physical or mathematical quantities. Difficulties arise also in relation to the units that have to be attributed to a concept. Do the dimensions of a concept have a reference in "reality"? Not only is it difficult to divide mathematical from physical quantities but it is also difficult to attribute to a physical quantity its dimensions; Planck and Sommerfeld in fact are in complete and explicit disagreement over this. As we will see below, Planck assumes as fundamental to his derivation of electromagnetic laws the Principle of Conservation of Energy and the Principle of Contiguous Action. This leads him to the following three expressions: for the density of electrical energy: $(\varepsilon/8\pi)\underline{E}^2$; for the density of magnetic energy: $(\varepsilon/8\pi)\underline{H}^2$; and for the flow of energy: $\underline{S}=(c/4\pi)\ (\underline{E} \times \underline{H})$ where e is the dielectric constant and m the magnetic permeability.

Thus, of the five quantities involved ($\underline{E}$, $\underline{H}$, ε, μ, c), two have to be defined arbitrarily. Planck shows that the different possibilities are five. At the end of the discussion about which should be preferred and on what grounds, Planck expresses an extremely interesting point of view on the problem of what a 'physical' quantity is:

> "The fact that when a definite physical quantity is measured in two different systems of units it has not only different numerical values, but also different dimensions has often been interpreted as an inconsistency that demands explanation, and has given rise to the question of the 'real' dimensions of a physical quantity. After the above discussion it is clear that this question has no more sense than inquiring into the 'real' name of an object." (32)

Quite opposite to this is Sommerfeld's approach in the preface to his Electrodynamics:

> "The dimensional character of the field entities is taken seriously throughout. We do not accept Planck's position, according to which the question of the real dimension of a physical entity is meaningless; Planck states in para. 7 of his Lectures on Electrodynamics that this question has no more meaning than that of the 'real' name of an object. Instead, we derive from the basic Maxwell equations the fundamental distinction between entities of intensity and entities of quantity, which has heretofore been applied consistently in the excellent textbooks of G. Mie. The Faraday-Maxwell induction equation shows that the magnetic induction B is an entity of intensity along with the electric field strength E; B, rather than H, deserves the name magnetic field strength."

and:

> "Energy quantities always take the form of products of an entity of quantity and an entity of intensity, e.g. $\frac{1}{2}\,\underline{D}\cdot\underline{E}$, $\frac{1}{2}\,\underline{H}\cdot\underline{B}$, $\underline{J}\cdot\underline{E}$, $\underline{E}\times\underline{H}$."

and:

> "These questions of units, dimensions, and rationalisation, often discussed to excess in recent years, are disposed of as briefly as possible in the lectures; however the reader is repeatedly urged in them to convince himself of the dimensional logic of the formulas."

and:

> "We may indicate finally a subdivision of physical entities into entities of intensity and entities of quantity. $\underline{E}$ and $\underline{B}$ belong to the first class, $\underline{D}$ and $\underline{H}$, to the second. The entities of the first class are answers to the question 'how strong', those of the second class, to the question 'how much'. In the theory of elasticity, for example, the stress is an entity of intensity, the corresponding strain, one of quantity, in the theory of gases pressure and volume form a corresponding pair of entities. In $\underline{D}$ the quantity character is clearly evident as the quantity of electricity that has passed through; in $\underline{H}$ the situation is slightly obscured by the fact that there are no isolated magnetic poles. We are in general inclined to regard the entities of intensity as cause, the corresponding entities of quantity as their effect." (33)

Sommerfeld states, following Hertz, Maxwell's equations as the starting point of the theory, and deduces all phenomena from them: "The totality of electromagnetic phenomena is derived from them systematically by deduction" and: "In agreement with Hertz we see in Maxwell's equations the essence of his theory." (34) The assumption of the equations and of Hertz's interpretation of them implies of course a stress on fields, as distinct from potentials, a "pre-established harmony" between the two, and a physical field with a finite propagation of the interactions. What is more, Sommerfeld stressed at a methodological level the importance of the rejection of induction from experiments, and of the mechanical approach to the equations: "We need not discuss the mechanical pictures, which guided Maxwell in the setting up of his equations." (35) But this is not the only possible approach to the problem of the interpretation of c. Planck shows (36) that even while rejecting induction and the mechanical pictures, the equations can be deduced from more general principles: the Principle of Conservation of Energy (PCE) and the Principle of Contiguous Action. This is a point of great methodological importance: contiguous action is seen as a model of greater heuristic power, and not as 'real' property of nature, attributed to the 'real field'. The finite speed of propagation acquires in this context the role of the modern expression of objectivity in physical laws (37). That is, it is now a necessary condition for the expression of a more specific, simpler law of interaction; it is not a sufficient condition for the existence of a "real" field. Thus both a naïve realist and an instrumentalist interpretation of contiguous action and time delay are superceded. In Planck the acknowledgement of greater heuristic power of a realistic interpretation of fields, goes together with the greater specificity of the theory and with its objectivity. But this objectivity is codetermined by the scientists' interpretation of what is more specific and by the physical phenomenology itself. The supersession of a direct theory - observation relation tends towards a realism depending in this way on heuristics, where the role of the theory is relevant in the expression of the reality.

C) Regulative principles component

With regard to the role of regulative principles in CET I will refer sometimes to the Principle of Least Action (PLA) and mainly to the PCE. The relations between PLA and PCE are extremely interesting and important for my analysis. In fact the sharp distinction between kinetic energy T and potential energy U, where the first depends on square velocity and the second on positions only, is not a necessary condition for either PCE or PLA. a) I shall describe, first, a potential which does not depend on positions only but also on velocities and b) second, a local principle of conservation. "Local" means that the transfer of energy is supposed to require time and thus its behaviour approximates that of matter. Thus the point of the first part of this section is to show that there are in CET textbooks different interpretations of the meaning of the PCE. Finally c) attention will be dedicated to the specific PLA used in the derivation of the force and fields equations

a) The classical principle of conservation of energy: d(V+T) = O is usually obtained as a special case of d'Alembert's principle:

$$\sum_s (\underline{F}_s^{(a)} + \underline{I}_s) \cdot \delta'\underline{r}_s = 0$$

where are applied forces, Isù are forces of inertia and w'rsù a reversible infinitesimal virtual displacement, (s = 1, 2, ... N),

if $\delta'\underline{r}_s = \dot{\underline{r}}_s dt$

(that is, if the virtual displacements coincide with an actual displacement of the system in time dt) we have:

$$\sum_s \underline{F}_s^{(a)} \cdot d\underline{r}_s - \frac{d}{dt}\left(\sum_s \frac{1}{2} m_s \dot{\underline{r}}_s^2\right) dt = 0$$

and if

$$\underline{F}_s^{(a)} = -\nabla_s \underline{V}$$

this gives the PCE in the form d(V+T)=0.

The condition that the work $\sum_{s=1}^{N} \underline{F}_s \cdot d\underline{r}_s$ may be expressed as a perfect differential -dV, where $V=V(r_i)$ is a function of the position coordinates riù, is often referred to as the condition for a Conservative system (38). This means that the work done by the forces is independent of the actual paths followed by the particles, and depends only upon their initial and final positions. But despite this being the standard presentation of the derivation of a mechanical conservation of energy, strictly speaking this definition of a conservative system is not correct. The problem concerns the function V, the potential energy. Must potential energy depend only on position? Whittaker more carefully speaks of initial and final configurations of the system (39). In the case of V=V(q) he speaks of a potential energy (where qiù are the generalised coordinates).

But he explicitly asserts (40) that in "certain cases the conception of a potential energy function can be extended to dynamical systems in which the acting forces depend not only on the position but on the velocities and accelerations of the body". In this case the generalised potential function is $U=U(q_i, \dot{q}_i)$. That is the potential depends on velocities as well as on position and cannot be sharply divided from the kinetic energy. But still in this case a Lagrangian function (kinetic potential) L=T-U exists and even if the system is not conservative in the usual sense the generalised forces can be obtained from a Lagrangean formulation:

$$Q_i = -\frac{\partial U}{\partial q_i} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_i}\right)$$

Goldstein's textbook specifies (41) that "the possibility of using such a 'potential' is not academic, it applies to one very important type of force field, namely the electromagnetic forces on moving charges." Actually Lorentz's force $\underline{F} = q\left\{\underline{E} + \frac{1}{c}(\underline{v} \times \underline{B})\right\}$ is derived from a Lagrangian equation in all the textbooks. But the interesting point for our purposes is a different one: that is, the reference to Weber's force. Goldstein recalls the history of the designation of generalised potential:

> "Apparently spurred by Weber's early (and erroneous) classical electrodynamics, which postulated velocity-dependent forces, the German mathematician E. Schering seems to have been the first to attempt seriously to include such forces in the framework of mechanics, eg. G|tt. Abh. 18, 3 (1873). The first edition of Whittaker's Analytical Dynamics (1904) thus refers to the potential as 'Schering's potential function', but the name apparently did not stick, for the title was dropped in later editions. We shall preferably use the name 'generalised potential', including within this designation also the ordinary potential energy, a function of position only." (42)

In fact, in Whittaker’s textbook, the example of generalised potential is not referred to Lorentz’s force but to Weber’s attraction law (43), even in the 1937 fourth edition; the function U is: $U = \frac{1}{r}\left(1 + \frac{i^2}{c^2}\right)$.

It is explicitly to be noted for the understanding of the relation between ’extraordinary’ and ’normal’ science, that it was the original paper of Weber in 1846 which initiated an extension of the idea of mechanical conservation; Whittaker again:

> "... Weber’s electrokinetic energy, which resembled kinetic energy in some respects and potential energy in others, could not be precisely classified under either head; and its introduction, by helping to break down the distinction which had hitherto subsisted between the two parts of the kinetic potential (Lagrangian function), prepared the way for the modern transformation theory of dynamics." (44)

The problem is just that in more modern textbooks these historical aspects of the problem of the relations between kinetic and potential energy are lost, and this usually implies a wrong judgment of some historical results as well as a bad starting-point for the historian’s work. Whittaker (45) quotes a Helmholtz paper of 1886 as reference for the general conditions for the existence of a kinetic potential of forces. This paper reveals that the problem was well known at the time, even though today the standard definition of mechanical energy conservation refers to a sharp distinction between kinetic and potential energy.

In order to assess the problem from a general modern point of view in what follows, I will refer to Lanczos’ position:

> "The quantity of prime importance for the analytical treatment of mechanics is not force, but the work done by impressed forces for arbitrary infinitesimal displacements. Particularly important for the variational treatment are forces which are derivable from a single scalar quantity, the work function U. Such forces may be called ’monogenic’. If the work function is time-independent, we obtain a class of forces called ’conservative’ because they satisfy the conservation of energy. In the frequent case where the work function is not only time - but also velocity - independent, the negative of the work function can be interpreted as the potential energy. Forces which are not derivable from a work function can still be characterised by the work done in a small displacement, but they do not yield to the general minimizing procedures of analytical mechanics."

and:

> "It is possible that the two basic quantities of mechanics, the kinetic energy and the work function, contain the time t explicitly. This will happen if some of the given kinematical conditions are time-dependent. It will also happen if the work function is an explicit function of the time as well as of the position coordinates (and perhaps of velocities). If both the kinetic energy and the work function are scleronomic, i.e. time-independent, the equations of motion yield a fundamental theorem called the law of conservation of energy. If either kinetic energy or work function or both are rheonomic, i.e. time-dependent, such a conservation law cannot be found." (46)

The importance of these general definitions will be clear in the following chapters.

b) In the case of continuous systems and of fields the energy conservation law is a local one, an energy continuity equation. The electric and magnetic field densities in the electromagnetic case are combined in the electromagnetic energy density of the field. This is a new step, for in the static case (stationary currents) the value of the energy is:

$$U = \frac{1}{2} \int \rho\phi dV + \frac{1}{2} \int \underline{J} \cdot \underline{A} dV,$$

where the electrostatic part was referred to as potential and the magnetostatic part as kinetic. Of course this is a mechanical action-at-a-distance with infinite speed approach. In the electromagnetic case of finite velocity of propagation of interactions, the two parts cannot be divided, just as in Weber's case (47). Now we have to speak of the electromagnetic field energy without distinction of kinetic and potential:

$$U = \int \left(\frac{\varepsilon_0}{2} \underline{E} \cdot \underline{E} + \frac{\varepsilon_0 c^2}{2} \underline{B} \cdot \underline{B} \right) dV,$$

In fact (48) "Within this larger context the Lagrangean density need not be given as the difference of a kinetic and potential energy density".

The use of PCE connected with a contiguous action theory is basic in Planck's approach (49). He starts from a regulative principle, PCE; he specifies this principle through the use of a second principle, the Principle of Contiguous Action (PCA); he very clearly states that the requirement of PCA defines the PCE in a more specific way than could be possible through the Principle of action-at-a-distance, but still refers to the specific form of the local principle of conservation of energy of Poynting as an experimental result. In Planck's words the "necessary uniformity and completeness" of an account of electromagnetic phenomena "can be achieved, it seems, only by using a predominantly deductive form of treatment" and:

> "Since among all the laws of physics none is so universal and so easy to grasp as the Principle of Conservation of Energy, I have again placed it in the forefront of discussion"

and:

> "this principle taken alone does not, of course, give us a sufficient hold to link up with it a definite theory of electricity. Rather, in the course of time several theories, all of which rest on the energy principle, have competed with each other ... The characteristic feature of the theory which is presented in this book and which was founded by Maxwell is given by a second fundamental idea: that of the Principle of Contiguous Action (Action by Contact)."

and:

"Since this proposition essentially restricts the possible ways in which physical causes can take effect, the principles of action-at-a-distance and of contiguous action ('far' action and 'near' action) are by no means to be regarded as coordinated; rather the principle of action-at-adistance is of a more general nature, whereas that of a contiguous action is rather more special. It is owing to this that there have been several different theories of action-at-a-distance in electrodynamics, but only one of contiguous action - namely, that of Maxwell. This theory, then, owes its sovereign position over all others not to its greater 'correctness', but rather to its greater definiteness and simplicity. For it states that when we wish to make calculations about events at a certain place we need not, in principle, take account of what occurs at another place which is at a finite distance away, but need only restrict ourselves to a consideration of the events that occur in the immediate vicinity. If we admit the principle of action-at-a-distance, on the other hand, we are, strictly speaking, compelled to search throughout the whole universe to see whether the events to be calculated are not influenced directly to an appreciable extent by an event at some other place. This, again, confirms the statement that the more special a theory is, and not the more general, the more definite are the answers which it gives to all fundamental questions and the more it will fulfill its proper task, which surely consists in making an unambiguous assertion about the phenomena that are expected to occur. This is a point which is unfortunately often overlooked in theoretical speculations. The fewer the undetermined constants that a theory contains, the more it achieves." (50)

Planck opens his analysis by stating that "it is not so easy to define the absolute value of the electric intensity of field. To arrive at it we start out from the concept of the energy of the field." The electrical density of energy is defined through the value $(\varepsilon/8\pi)\ \underline{E}^2$ and the magnetic one through $(\mu/8\pi)\ \underline{H}^2$.

According to the principle of conservation of energy, the electromagnetic energy of the part of a homogeneous body which is permanently at rest and in which there is an electromagnetic field can be changed only if there is an exchange of energy with outside bodies. The principle of contiguous action restricts the possible exchanges of energy with the surroundings to those regulated by a flux of energy, analogous to the flow of a fluid, through the surface of the enclosed space. The energy flux is called $\underline{S}$. Now Planck comes to the specifications of PCE through contiguous action: this specification is not univocally defined but needs the reference to 'experiences':

"The manner in which the energy flux S depends on the field-strengths $\underline{E}$ and $\underline{H}$ must be deduced from experience. It has been shown to be regulated by a very simple law which we take as our main pillar in building up the electromagnetic field equations, since it is the most comprehensive and exhaustive expression of all the experimental results gathered in this sphere. This law is Poynting's law of Energy Flux, which states that $\underline{S}$ is proportional to the vector product of $\underline{E}$ and $\underline{H}$, that is:

$$\underline{S} = \frac{c}{4\pi} \cdot \underline{E} \times \underline{H}$$

"Although these relations have the disadvantage that we cannot easily visualise them pictorially, this is more than counterbalanced by the fact that from them, as we shall see later, definite quantitative laws can be derived for all electric and magnetic processes in a simpler manner without introducing any particular experimental results." (51)

c) A second regulative principle often used in CET textbooks is PLA. Since Schwarzchild's 1903 paper, from PLA both Maxwell's equations and Lorentz's force can be derived. But is it possible to derive these laws without reference to the potentials? On the relations among fields, potentials, variational principles and energy expressions Lanczo's position is very clear:

> "The Maxwellian scheme operates solely with the electric and magnetic field strengths E and B, and the question can be raised whether these equations in the given form might not be derivable from a variational principle, without reference to the vector four-potential. This, however, cannot be done in a natural way, since we cannot expect to obtain eight equations by varying six quantities." (52)

Thus the variational derivation needs the potentials. Are they to be considered mathematical or physical quantities? And moreover, which is the energetic interpretation of the terms in the variational formulation? Landau's and Lipschitz's approach is summarised below, but it must be emphasised that whatever the answer, a sharp distinction between kinetic and potential energy is definitely lost in electromagnetism. Both Weber's and Lorentz's forces, Poynting's theorem, and the use of scalar and vector potentials imply a departure from the old mechanical interpretation of energy.

Landau assumes a PLA as the basis of his exposition of CET, where the total action is:

$$S = S_m + S_{mf} + S_f = - \Sigma\, mc \int ds - \Sigma \int \frac{e}{c} A_k\, dx^k - \frac{1}{16\, \pi c} \int F_{ik} F^{ik}\, d\omega$$

where S_m is the part of the action depending only on the properties of the particles (m is the mass and c the light velocity), S_{mf} on the interaction of the particle with the field (A_k are the potentials and e the charge), and S_f on the field itself considered in absence of charges (S_f is considered to depend on electric and magnetic densities (through the tensor F_{ik}) and not on potentials). From these equations, with the help of the Principle of Superposition, and a few other considerations, Maxwell's equations and the expression of Lorentz's force are derived. But it is interesting to note that the starting point, i.e. the definition of the term action = S_{mf}, presents a certain degree of arbitrariness, solved by Landau and Lipschitz with a reference to 'experimental results' not better specified:

> "Il apparait que les propriétés de la particule sont définies, en ce qui concerne son interaction avec le champ, par un seul parametre, appelè charge e de la particule, pouvant étre positif, négatif ou nul. Les propriétés du champ, elles, sont caracterisèes par un 4-vecteur A_i, appelè 4potential, dont les composantes sont des fonctions des cordonnèes et du temps. Ces grandeurs sont exprimèes dans l'action au moyen du terme
>
> formula
>
> où les fonctions Aiù sont prises le long de la ligne d'univers de la particule."

and:

"Les affirmations faites çi-dessus doivent Ìetre considerèes, dans une bonne mesure, comme le resultat de donnèes expèrimentales. La forme de l'action pour une particule dans un champ èlectromagnètique ne peut Ìetre ètablie seulement a partir de considèrations gènèrales, telle la condition d'invariance relativiste." (53)

D) Experimental component

Both the 'experimental results' of Landau-Lipschitz and of Planck are not at all easily recognisable as experimental. In the case of Poynting the historical difficulties of the deduction of his result are referred to in the chapter on "The Eighties". From a modern point of view, moreover, there are still problems with an exact specification of Poynting's vector (54). The previous analysis outlines a circularity in the presentation of CET in textbooks: the old naive inductive approach based on the 'experimental results' of Coulomb, Oersted, Ampère, Faraday, Hertz etc. has been superseded by a more sophisticated approach in which the regulative principles play a great role. But still, the experimental results are recalled for a more precise and necessary specification of the principles themselves. The notion of cruciality of the experiments does not find a place in Nobel prize winners' textbooks, but still a theory-experiment interplay exists and does have an important role. In agreement with recent epistemological analysis (55) a clear distinction between theory and observations cannot be drawn in principle, but may be drawn by referring to scientific practice. A problem resides in the obvious difficulties of expressing this interplay.

My analysis does not deal with this problem here. I analysed somewhere else the case of Hertz's experiments to show their lack of "cruciality" (46). What is relevant here is to have shown that in CET textbooks there is a) an interplay of four components; b) a varying evaluation of the role of the single components. The present work deals mainly with the interplay of the component of regulative principles with the component of conceptual models. In my view the results of the analysis show the CET debate in a new light. The points outlined above show that it is not possible to define a 'standard' classical exposition of CET. The Nobel prize winners' textbooks are different in important respects and none of them can be considered superior to the others. This is one of the cases in which a closer look at 'normal science' textbooks shows that this notion loses its validity in a more specific range of approximations.

Part II

Indeed the dynamic interplay of the four components in the historical development of CET offers even more interest than the static one in the modern assessment. And again, the specific use by the scientists of each component shows interesting physical aspects as well as philosophical ones. The historical research is an application of the above approach to the debate on electromagnetic phenomena in the second half of last century. It starts from 1845 with the publication of Neumann’s law and ends at the beginning of the twentieth century, with Schwarzchild’s derivation of the force and field laws (1903). There were two main contrasting conceptual models, contiguous action and action-at-a-distance, both having their roots in the mechanical world view: the first refers to the mechanics of continuous bodies, and the second to that of particles. At the end of the nineteenth century, neither of them could be said to have won the day: Lorentz’s theory represents a synthesis, sharing elements of both approaches. The settling of the debate was obviously temporary; as was shown, attempts are still being made to build an electromagnetic theory based only on one or the other of the two fundamental models. In this work, these later developments will not be dealt with. A quotation in my view illustrates the usefulness of the four-component approach. It is from Maxwell’s Treatise:

"In a philosophical point of view, moreover, it is exceedingly important that two methods should be compared. Both of which have succeeded in explaining the principal electromagnetic phenomena, and both of which have attempted to explain the propagation of light as an electromagnetic phenomenon and have actually calculated its velocity, while at the same time the fundamental conceptions of what actually takes place, as well as most of the secondary conceptions of the quantities concerned, are radically different." (47)

Now we see how the four components come into play: the contrasting conceptual models gave equivalent contributions, their mathematical formulation was equivalent, the experiments could be often explained in most different theories; on what grounds did the debate develop? My approach also outlines the role of the second component in the classification above i.e. the regulative principles, and mainly of PCE, in this debate. Of course it would be too naïve to expect to find the main or the only reason for the settling of the debate, but neverthless such an approach shows a set of interesting points which are usually underrated and casts a new light on the relations between PCE and CET. In fact, what will be shown is the role played by this principle as a regulative device in the construction of CET theories, its initial use when linked with a mechanical conception and its consequent detachment from this, and the different interpretations it has received in the different schools. The extension and the length of the debate are also impressive: most of the scientists of the European countries took part in this confrontation during the nineteenth century. In fact although my analysis is restricted to the period from 1845 to 1903 (from F. Neumann's potential law to K. Schwarzchild's variational derivation of the force and field equations) at least twenty famous scientists took part in this debate. They are: W. Weber, F. Neumann, R. Kohlrausch, G. Kirchhoff; K. Gauss, B. Riemann, C. Neumann, L. Lorenz; H. Helmholtz, R. Clausius, H. Hertz, M. Planck; W. Thomson, J.C. Maxwell, W. Rankine, J. Poynting, J.J. Thomson; H. Lorentz; H. Poincarè; K. Schwarzschild. A chronological table of the main papers analysed is listed below, where the authors are divided into six different groups: the German school of action-at-a-distance; the German school of action-at-a-distance with retarded potentials; Helmholtz, Clausius, Planck and Hertz; the British school of contiguous action; H. Lorentz; Poincarè.

Chart 1

	ACTION AT A DISTANCE	RETARDED POTENTIALS	HELMHOLTZ, CLAUSIUS, HERTZ, PLANCK	CONTIGUOUS ACTION	H. LORENTZ	POINCARE
1845	F. Neumann F_1	Gauss		W. Thomson		
1846	W. Weber F_2					
1847			Helmholtz PCE_1	W. Thomson		
1848	F. Neumann, W. Weber					
1849	Kirchhoff, Kohlrausch			W. Thomson		
1850				W. Thomson		
1851						
1852	W. Weber		Clausius			
1853				Rankine, W. Thomson		
1854			Helmholtz a,b			
1855				Rankine		
1856				Maxwell		
1857	Weber e Kohlr.,Kirchh. a,b					
1858		Riemann (publ. 67)				
1859			Clausius	Rankine PCE_2		
1860				W. Thomson		
1861	Riemann	L. Lorenz	Helmholtz			
1862				Maxwell		
1863						
1864	Weber					
1865				Maxwell		
1866						
1867		L. Lorenz		Thomson e Tait, Rankine		
1868		C. Neumann		Tait		
1869						
1870			Helmholtz F_3			
1871	Weber PCE_3	C. Neumann				
1872			Helmholtz			
1873		C. Neumann	Helmholtz a,b	Maxwell F_4		
1874		C. Neumann				
1875			Clausius F_5, Helmholtz		Lorentz	
1876			Clausius PCE_4			
1877		C. Neumann				
1878					Lorentz	
1879			Hertz, Clausius			
1880						
1881			Helmholtz			
1882						
1883						
1884	Hoppe		Hertz	Poynting PCE_5		
1885				Poynting, JJThomson a,b		
1886						
1887			Hertz, Planck PCE_6			
1888			Hertz			
1889			Hertz			
1890						Poincaré
1891					Lorentz	Poincaré
1892			Hertz		Lorentz F_6	Poincaré
1893						
1894			Helmholtz			Poincaré
1895					Lorentz	
1896						
1897			Helmholtz			
1898		C.Neumann, Lienard				
1899						
1900		Wiechert				
1901						Poincaré
1902						Poincaré
1903		Schwarzschild				

The chart starts (from top to bottom) with the Leibnizian ($PCVV_1$) and the Newtonian ($PCVV_2$) influence on Helmholtz (1847 - PCE_1). In fact Galileo's and Huygens' results on pendulum drew attention to the importance of the quantity mv^2 in connection with the principle of impossibility of perpetual motion. But the connection was realised in different ways, according to two aspects of the principle: the "ex nihilo nil fieri" and the "nil fieri ad nihilum". The first refers to the impossibility of destroying work. Leibniz focused his attention on the latter aspect and analysed the causal relations between static and dynamic phenomena. He related both to a common reference, work, and gave a numerical value to the causal relation. In this context, the Principle of Conservation of

Vis Viva means the conservation of a cause/effect relation between static and dynamic phenomena, i.e. the numerical equivalence of cause and effect measured in work units. The "ad nihilum nil fieri" here means that a given quantity does not disappear but is tranformed into a different but equivalent form. ($PCVV_1$)

The other tradition (the so-called Newtonian one), of D'Alembert, D. Bernoulli and Lagrange, focused on the "ex nihilo" aspect: vis viva was only meant as a function of position, and independent of constraints and trajectories of the bodies. The link between the internal (velocities) and external (forces and positions) aspects was analysed, and the variation of vis viva was correctly connected to the work done, through the vis viva theorem. Conservation here means conservation at a certain position, independently of the path followed to arrive at that position ($PCVV_2$). A second step was to develop a function of position from which the forces could be deduced: the potential. The existence of a potential was a new formulation of the "ex nihilo": for a closed path the work is zero. Despite the acknowledgement that the sum of vis viva plus the potential is a constant, no special attention was paid to that result.

At the beginning of the 19th century, the Mathematical Potential Theory (MPT) rapidly develops, mainly in connection with mechanics and electrostatics. At the same time, CET also develops, and at the beginning of the forties one of the problems was the unification of Coulomb's law for static charges, Ampère's law for closed currents and the Faraday-Lenz law of induction. This was the input for F. Neumann, who solved the problem in 1845 (F_1) applying the MPT and a concept of potential interpreted as the work done in moving charged conductors to the position they actually have. The main output was to be towards Helmholtz (1870) and Clausius (1875). In fact Clausius' potential is identical to Neumann's if currents are interpreted as charges in motion. A second unification was achieved by W. Weber in 1846 (F_2) through a force law. Weber introduced a main change in the concept of force: his forces depended not only on position but also on velocity and acceleration of charges. In 1848 he showed that the law could be deduced from a potential. Thus both F. Neumann and W. Weber were in agreement with the "ex nihilo nil fieri". Weber (F_2) influenced C. Neumann (1868), who showed that delayed action-at-a-distance is equivalent to assuming forces dependent on velocities and accelerations, as well as Helmholtz (1870) and in more modern times Feynman's reformulations of delayed action-at-a-distance.

Helmholtz deserves the credit for the unification of the two mentioned traditions: the principle of causality of the Leibnizian tradition and the model of force of the Newtonian one. In his paper of 1847 the impossibility of perpetual motion was transformed into the Principle of Conservation of Energy (PCE_1). The "ad nihilum" meaning allowed the generalisation of the conservation to all realms of physics, electromagnetism included. A necessary requirement for Helmholtz was the Newtonian model of central forces, and thus a strong polemic began against Weber's non-central forces. A necessary consequence of the model of force adopted is also Helmholtz's sharp distinction between two forms of energy, kinetic and potential. On the other hand, such a distinction is impossible if one adopts Weber's basic law. His paper is relevant to all four components and its output is so large that it was not graphically reproduced. one (the effect) (PCE_2). He was the first to utilise the term "potential energy" and his methodological contributions were important. His influence will appear in Maxwell's Treatise and much later in Sommerfeld's CET textbook.

Kirchhoff's (1849) unification of the concepts of tension and difference of potential had an influence on W. Thomson's (1854) "equation of telegraphy", which in turn was relevant to Kirchhoff's (1857) assessment of the law of propagation in conductors. Together with Weber's and Kohlrausch's (1857) determination of the ratio cwù between electrostatic and electrodynamic forces and Weber's (1864) interpretation of it as a wave velocity, these results at the beginning of the sixties settled the problem of the finite speed of propagation in conductors. potential energy. In fact, he showed the equivalence of surface with volume integrals of the electric and magnetic intensities. The surface integrals were referred to action taking place in the medium between conductors. W. Thomson's works of the fifties had two more relevant results: on one hand the laws of conductors influenced L. Lorenz's (1867) delayed action-at-a-distance approach, on the other, his demonstration of the equivalence between action-at-a-distance and contigous action stationary energy values was to play a major role in Maxwell's electromagnetic theory.

I postponed Maxwell's great contribution until his Treatise (1873); meanwhile, the sixties appear important mainly for the contributions of the delayed action-at-a-distance school. After some initial results of Gauss and Riemann, L. Lorenz and C. Neumann develop an alternative view of electromagnetic phenomena: the time delay in the interaction is accepted not only for conductors, but also for dielectrics and free space, but at the same time action-at-a-distance is retained. That is, Poisson's equation is transformed into d'Alembert's equation without a clear physical interpretation of the quantities involved. Difficulties in the comprehension of the concept of energy in this framework did not affect the formal equivalence between these results and the contiguous action equations of Maxwell's second paper (1864). Both approaches stressed the importance of an electromagnetic theory of light. Thus at the end of the sixties all the approaches had reached a main theoretical point: the delay of propagation of electromagnetic interactions, both inside and outside conductors. Also relevant was the distinction between static, stationary and variable electric conditions. The delay was in fact theoretically relevant only in this latter case, which was outside the range of existing experimental limits. L. Lorenz's retarded potentials found a lasting recognition in Schwarzschild's (1903) variational derivation.

The seventies are characterised by a deep theoretical struggle. In 1870 H. Helmholtz published a basic paper in which he set up a confrontation between the main existing theories. He succeeded in formulating a general potential law from which the different laws were deduced as specific cases. This formal correspondence did not mean equivalence of interpretations: in Helmholtz's approach, all the other theories were analysed in an actionat-a-distance framework. This paper had a considerable influence on the German scientists. It resulted in a long polemic between Helmholtz and Weber on the relations between force law and Principle of Conservation of Energy. Helmholtz firstly denied that Weber's force law was in agreement with PCE_1, and secondly showed some physical inconsistencies deriving from Weber's law. Here the role of PCE in the CET debate comes fully into play. In 1871 Weber in fact denied the contrast between his force law and PCE: he gave a new interpretation of PCE (PCE_3) and denied the validity of Helmholtz's PCE_1. The main point under debate was the acceptance or refusal of the sharp distinction between kinetic and potential energy. Acceptance meant the acceptance of Newtonian forces, refusal meant the acceptance of Weber's forces. The polemic started in 1847 and lasted for almost fifty years. Helmholtz's paper had a tremendous impact. It spread Maxwell's idea (reformulated in different terms) on the Continent and was a main input for Lorentz's early work and for Hertz from 1879 to 1888.

In 1873 Maxwell published his Treatise that was as well to be widely influential. His aim was to start from the action-at-a-distance results and to transform them into Faraday's view of contiguous action. To achieve that, Maxwell should have started from the electromagnetic actions, produced by the sources, charges and currents; he should have established their equivalence with the action taking place in the intervening medium, and then reconsidered these last actions as the only ones actually existing. At this stage, to accomplish a unitarian view he should have reconsidered the original sources, as the effects of the new sources, i.e. of the actions in the medium. But Maxwell stops in the middle. He does not accomplish the last step and thus does not establish a clear priority of the actions in the medium. Despite that, the equivalence he establishes is fundamental, and it is particularly important that he establishes this equivalence by considerations referring to energy. In fact it is W. Thomson's result mentioned above that is utilised by Maxwell as the formal ground for the conceptual shift from action-at-a-distance to contiguous action. In this shift an important role is played by the displacement current, i.e. by Maxwell's conception of all currents as closed currents. In Maxwell's view, the displacement current completes Ampère's law, and introduces a current in dielectrics that has the same electromagnetic effects as the conduction current. Moreover this electric displacement is precisely related to the value of energy in the medium.

Thus in Maxwell, there is a model of contiguous action in dielectrics (ether included), but this action (probably) still depends on charges and conduction currents which are considered to be the sources. Maxwell's equations (i.e. the Coulomb-Gauss, Faraday, and Ampère equations generalised) entail a propagation in time with a velocity equal to c and with transverse waves, but at this stage they are formally equivalent to the equations of delayed action-at-a-distance of Lorenz, as explicitly noted by Maxwell in the Treatise. The physical interpretation however is completely different. The main difference concerns the concept of energy: in Maxwell, energy is localised outside conductors in space. Later on, the abandonment of the idea of "displacement" will allow the electric and magnetic polarisation to acquire an autonomous status, independent of the idea of mechanical ether. At this later stage, the energy of electric and magnetic intensities will have priority with respect to the charges and currents. This step was to be accomplished by Poynting, but still Maxwell in the Treatise realizes something important for the localisation of energy: he assumes as a basic quantity the electrodynamic potential of two circuits (introduced by F. Neumann) and splits it into two terms. The first term is electrostatic energy (depending on the position of charges) and the second is electrodynamic energy (depending on intensities of currents). Moreover, Maxwell localises these two terms in space, identifying the first with potential (U) and the second with kinetic energy (T) of ether. This allows him to maintain the sharp distinction between T and U despite the abandonment of central forces, and allows him, with some restrictions, to apply a classic Lagrangian derivation to his system of linear conductors. Finally, in the last chapter of the Treatise, he analyses the action-at-a-distance theories, and considers the interpretations of energy terms as basic, because of the formal equivalence of the experimental results of the main theories. It is on the grounds of energy localisation that he claims his own theory be preferred. Maxwell's program had a deep heuristic power, but Maxwell himself did not fulfil it. His use of a mechanical distinction of kinetic and potential energy shows his reliance on action-at-a-distance concepts. In electromagnetism in fact this distinction holds only in the stationary case. But still, his first step of localisation and his idea of contiguous action (action at small distances) was to be widely influential.

Again, Clausius demonstrates the importance of the interplay between force law and PCE. From 1852 Clausius's approach to PCE became quite different from Helmholtz's: Clausius is more analytical, his reinterpretation of the vis viva theorem does not imply central Newtonian forces. In fact in 1875 he deduces a force law depending on velocities and accelerations, and in 1876 asserts that to satisfy PCE the only condition on the forces is that the work done is a complete differential. Thus he is thinking along the lines of the "ex nihilo nil fieri", but at variance with Weber, since he rejects the hypothesis that equal and opposite charges move with opposite relative velocities. Clausius utilises a non-classical Lagrangian derivation, where kinetic and potential energy are not sharply divided. His electrodynamic potential, retarded in the way of L. Lorenz, was to be part of Lorentz's theory and of Schwarzschild's derivation of 1903. Clausius had as input F. Neumann's F_1 through the mediation of Helmholtz's F_3. His result was influential on Lorentz's F6ù and found a place in Schwarzschild (1903).

Thus, at the end of the seventies, the electromagnetic debate was very lively. Its most important features appear to be the mathematical equivalence and the theoretical contrasts of the main conceptual models. The practical impossibility of an experimental comparison opened the way to a theoretical confrontation on the grounds of the regulative principle of conservation of energy. The "ex nihilo" line stressed the relevance of the existence of a potential function. The "ad nihilum" line (Maxwell) stressed the importance of a substantialisation of the conserved energy and initiated a theoretical interpretation of its localisation. Helmholtz, who in 1847 had unified the two lines in a mechanical framework, had now to face a problem: he had to give up the model of central forces. He could have done that in two different ways. The first would have been to accept Weberian forces and thus reject a sharp separation between kinetic and potential energy, the second, to accept contiguous action and maintain the sharp distinction between kinetic and potential energy. He preferred the second choice. The localisation of energy was to play an important role in that choice.

At the beginning of the eighties, the main schools were still competitors as far as the kind of propagation was concerned. In the eighties important results were defined: in fact in the nineties, most scientists shifted towards contiguous action. Against the usual historiographical approach that attributes the shift to Hertz's experiments of the late eighties and attributes to PCE and PLA only justificative value (if any), here evidence has been produced of the existence of various versions of PCE and of the relevance of Poynting's (1884) local principle of conservation for the shift. The physical difference between contiguous action and action-at-a-distance has to be referred to the range of actions (short and large) and not to the time-delay or to the dielectric. This difference appears clear in the energy concept. The localisation of energy and its flow are two steps connected with the idea of short range of interactions. While the first step in stationary conditions allowed an equivalence with action-at-a-distance, the second was possible only in a framework of contiguous action. Maxwell established only a formal equivalence with opposing theories, but produced a powerful heuristic program.

In fact in 1884 and 1885 Poynting completes Maxwell's theory of electromagnetic energy. In his theorem, energy flows continuously in space and time like fluid matter. Thus conservation is no longer global but local, time has to be considered as it is in a continuity equation (PCE_5). Another important result is to establish finally the priority of electric and magnetic intensities with respect to charges and currents: energy is no longer interpreted as carried along the wire, but as a propagation around it. Moreover, kinetic and potential energy in non-stationary conditions cannot be sharply divided: the abandonment of all sorts of mechanical ethers implied the rejection of this last mechanical condition. Maxwell's program was thus accomplished. PCE_5 was to be fundamental in Planck's 1887 analysis, and influential in the shift towards contiguous action of Hertz and Lorentz.

In 1885, J.J. Thomson carefully analyses all the formulations of CET in relation to their derivability from mechanical principles and to the acceptance or rejection of a dielectric. Again PCE plays a main role as ground of comparison for the different theories.

But Planck's treatise of 1887 is here considered as the real turning-point in the CET debate. In fact, with this work, the whole debate shifts onto a more sophisticated level. The equivalence of the interaction laws and the general fulfilment of PCE are acknowledged. Thus the object of the debate now is: which is the version of PCE to be preferred? Planck analysed in detail all the previous interpretations of PCE and gave a new general formulation that included the previous ones, as well as the two meanings of the "ex nihilo" and the "ad nihilum". In the section on electromagnetic phenomena, Planck applied his PCE to the analysis of the debate. The comparison of PCE's was made on heuristic grounds, i.e. the theory that was to allow greater testable predictions was to be favoured. He judged that the localisation of energy obtainable through contiguous action had great heuristic value and had to be preferred over the action-at-a-distance approaches for its greater specificity rather than for its "correctness". This judgment, prior to Hertz's experiments, was the rationale for the shift towards Maxwell's contiguous action that was largely accomplished in the nineties. It is reproduced in Planck's (1932) textbook.

Hertz's experiments ('87-'90) are here considered important but not crucial for the CET debate. The three series of experiments are recalled, in relation to Hertz's theoretical interpretation. His shift from action-at-a-distance to contiguous action happened in 1888 during the second series of experiments, which dealt with the finite velocity of propagation. Actually Hertz did not directly prove the existence of the displacement current, and thus no "crucial" value can be given to his second series of experiments (the finite velocity of propagation was predicted by delayed action-at-a-distance also). More relevant for the acceptance of contiguous action was the third series of experiments: but again the analogies with the behaviour of light are not considered here to be crucial. What was supposed to be proved at the time was the existence of the ether. Planck had distinguished between time delay, contiguous action and existence of a substantial ether. Neither time delay (proved by Hertz), nor existence of a substantial ether (supposed to be proved by Hertz in the late eighties but refused in the early nineties) were possible grounds for a decision between the different approaches (both were predicted by the opposite schools).

In addition, a careful analysis of Hertz's results required some years, while the shift towards contiguous action was almost contemporary. Hertz's axiomatic presentation (1892) was to be influential on Sommerfeld's textbook, while Pauli's one is more connected with an experimental "inductive" approach.

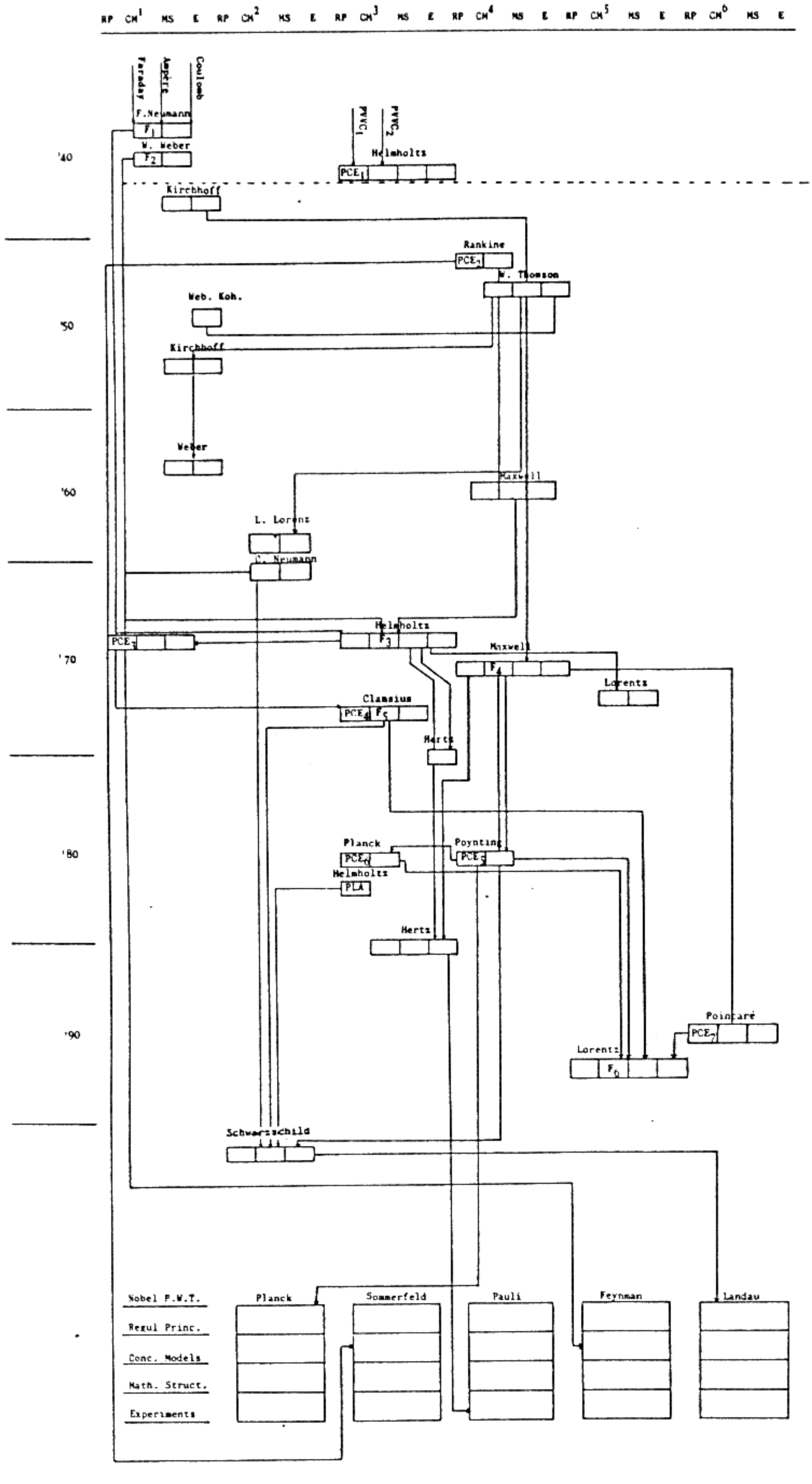
In Poincarè's long analysis of the CET debate, one point is chiefly stressed: his preference for Maxwell's contiguous action. The reason for this preference is the localisation of energy that allowed a sharp distinction of T and U and the use of a classical Lagrangian derivation. Without this distinction of T and U, and their localisation, in Poincarè's view, no specific meaning could be attributed to the principle of conservation: "something is conserved", but we do not know what it is.

Lorentz’s synthesis shared elements of both the British and the German traditions. His preference for contiguous action was again explicitly based on energy considerations. Finally, Schwarzschild in 1903 deduced both the force law (Lorentz’s law derived from a retarded Clausius’ potential) and the field from an action-at-a-distance point of view, and this showed again that the two models were formally equivalent. Schwarzschild’s variational derivation used retarded potentials that did not permit a direct energetic interpretation. But one point has to be stressed here: Schwarzschild’s derivation had justificative but not heuristic power, since all the results he reformulated were well-known. Schwarzschild (1903) summarizes several results (see flow chart): the F. Neumann-Clausius potential retarded through L. Lorenz’s approach; Helmholtz’s (1887) approach to a variational derivation; Maxwell-Poynting’s results of contiguous action and local conservation. This was to be basic in Landau’s textbook.

Despite the formal equivalence of the two models throughout the whole debate, the main point of my work is that the localisation of energy was favoured in the late eighties explicitly (in Planck) for its heuristic power.

A flow chart summarises the main problems analysed in this work.

Chart 2



RP CM1 MS E RP CM2 MS E RP CM3 MS E RP CM4 MS E RP CM5 MS E RP CM6 MS E
Faraday
Ampère
Coulomb
F. Neumann
W. Weber
Helmholtz
Kirchhoff
Rankine
W. Thomson
Web. Koh.
Kirchhoff
Weber
Maxwell
L. Lorenz
C. Neumann
Helmholtz
Maxwell
Lorentz
Clausius
Hertz
Planck
Poynting
Helmholtz
Hertz
Poincaré
Lorentz
Schwarzschild
'40
'50
'60
'70
'80
'90
Nobel P.W.T.
Regul Princ.
Conc. Models
Math. Struct.
Experiments
Planck
Sommerfeld
Pauli
Feynman
Landau

The connections between the results of different authors show at the same time a principle of correspondence of quantitative results with previous theories and the translation of meanings of the same formal results in the new conceptual models.The chart is divided into six vertical columns representing the different schools (as in the chronological table). Each column is divided into the four components. The papers are represented with sets of little squares corresponding to components. Only a few papers and a few connections between them are mentioned in the chart for graphical reasons. Thus the chart is meant to be only a qualitative overall view. It is mainly directed at framing the interaction laws (F) of the different schools and the corresponding versions of the Principle of Conservation of Energy (PCE). A list of the formal laws and of the formal expressions of the principles is outlined.

List

	GALILEO	$v = \sqrt{2gh}$	
	HUYGENS	$\Sigma mv^2 = \Sigma mu^2$	
1686	LEIBNIZ	mv^2 = vis motrix; Imp. perp. mot. → Σmv^2 = cost.	
1724	J. BERNOULLI	cause = eff.	$PCVV_1$
1743	D'ALEMBERT	$\int_1^2 \underline{F}\cdot d\underline{s} = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$	
1748	D. BERNOULLI	$\Sigma mv^2 = \Sigma mx \quad (x=2h;\ g=1)$	
1811	LAGRANGE	$\Sigma m\ (\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2 + 2\pi = 2H$	$PCVV_2$
1847	HELMHOLTZ	$\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 = \int_1^2 \underline{\phi}\cdot d\underline{s} \rightarrow T+U$ = cost.	PCE_1
1853	RANKINE	$T = (\frac{mv}{t})(\frac{1}{2}vt);\ U = (\frac{Fmm'}{ar_1})(r_1 - a)$	PCE_2
1871	WEBER	$U + x = a$	PCE_3
1875	CLAUSIUS	$\frac{1}{2} m_s {v_s}_2^{\,2} - \frac{1}{2} m_s {v_s}_1^{\,2} = \int\Sigma(Xdx + Ydy + Zdz) = \int\Sigma dL$	PCE_4
1884	POYNTING	$\underline{S} = \frac{\text{el int x magn int x sin incl. angle}}{4\pi}$	PCE_5
1887	PLANCK	$\int dL_1 + \int dL_2 + \int dL_3 + \dots = E_1 - E_2$	PCE_6
1890	POINCARE'	$E = f(T+U)$	PCE_7
1785	COULOMB	$\underline{F} = k\, e_1 e_2 \frac{(\underline{x}_1 - \underline{x}_2)}{\lvert\underline{x}_1 - \underline{x}_2\rvert^3}$	
1820	OERSTED-BIOT SAVART	$\bar{B}(1) = \frac{i}{r_{12}^3}\, d\bar{s}_2 \times \bar{r}_{12}$	
1825	AMPERE	$\bar{F} = -cc'\underline{r}\ (\frac{2}{r^3}(d\underline{s}\cdot d\underline{s}') - \frac{3}{5}(d\underline{s}\cdot\underline{r})(d\underline{s}'\cdot\underline{r}))$	
1826	OHM	$S = \gamma E$	
1834	FARADAY LENZ	$E = -k\frac{d\phi}{dt}$	
1845	F. NEUMANN	$M = ii'\int_s\int_{s'}\frac{d\bar{s}\cdot d\bar{s}'}{r}$	F_1
1846	W. WEBER	$F = \frac{ee'}{r^2}(1 \pm \frac{1}{c_w^2}(\frac{dr}{dt})^2 + 2\frac{r\,d^2r}{c_w^2 dt^2}) \quad U = \frac{ee'}{r}(\frac{1}{2c_w^2}\cdot\frac{dr^2}{dt^2} - 1)$	F_2
1857	KIRCHHOFF	$\bar{E} = \text{grad}\,\phi - \frac{1}{c}\frac{\partial\bar{A}}{\partial t}$	
1867	LORENZ	$\phi = \iiint(e'(t-r'/c)/r')dx'dy'dz' \quad \bar{A} = \frac{1}{c}\iiint(\bar{j}'(t-r'/c)/r')dx'dy'dz'$	
1870	HELMHOLTZ	$M = \frac{jj'(d\bar{s}\cdot d\bar{s}')}{r} + kjj'\frac{d^2r}{dsds'}dsds'$	F_3
1873	MAXWELL	$\bar{B} = \text{rot}\,\bar{A};\ \bar{E} = -\frac{\partial\bar{A}}{\partial t} - \text{grad}\,\phi;\ 4\pi\bar{C} = \text{rot}\,\bar{H};\ \bar{C} = \bar{J} + \frac{\partial\bar{D}}{\partial t};\ \bar{D} = \frac{K}{4\pi}\bar{E}$	
		$\rho = \text{div}\,\bar{D};\ \bar{B} = \mu\bar{H}$	F_4
1875	CLAUSIUS	$F_x = -\frac{\partial 1/r}{\partial x}(1 - Kvv'\cos\ ) - k\frac{d}{dt}(\frac{1}{r}\frac{dx'}{dt});\ M = \frac{ee'(\bar{v}\cdot\bar{v}')}{r} +$	
		$+ \frac{Kee'\,d^2rvv'}{dsds'}$	F_5
1890	HERTZ	$\frac{1}{c}\frac{\partial\bar{H}}{\partial t} = -\text{rot}\,\bar{E};\ \text{div}\,\bar{H} = 0;\ \frac{1}{c}\frac{\partial\bar{E}}{\partial t} = \text{rot}\,\bar{H};\ \text{div}\,\bar{E} = 0$	
1892-5	H.A. LORENTZ	$F = e\bar{d} + \frac{e}{c}(\bar{v}\times\bar{h});\ \text{div}\,\bar{D} = 4\pi\rho;\ \text{div}\,\bar{B} = 0;\ \text{rot}\,\bar{H} = \frac{4\pi}{c}\bar{S};$	
		$-\text{rot}\,\bar{E} = \frac{1}{c}\frac{\partial\bar{B}}{\partial t};\ \text{div}\,\bar{S} = 0$	F_6
1898-1900	LIENARD-WIECHERT	$\phi(1,t) = q/4\pi\epsilon_0\,[r-(\bar{v}\cdot\bar{r}/c)]_{ret};\ A(1,t) = qv/4\pi\epsilon_0 c^2[r-(\bar{v}\cdot\bar{r}/c)]_{ret}$	
1903	SCHWARZSCHILD	$\int dt(-T+\Sigma eL);\ L = \phi - v_x\Gamma_x - v_y\Gamma_y - v_z\Gamma_z;\ \int dt\,dw(\frac{H^2-K^2}{8\pi} + XL)$	

The flow chart thus summarises the main points analysed. The chart is divided into two main sections: the upper part is historical (1845-1903) and the lower part is "logical", i.e. there are no historical connections between the five textbooks. The two parts are in my view indivisible and there is not only one direction of the flow. A full historical understanding of the development of CET requires a preliminary analysis of contemporary textbooks, i.e. an explicit pattern from textbooks to original papers. A full logical understanding of contemporary CET requires a knowledge of its historical roots. A full understanding of both the extraordinary and normal aspects of science requires a process of increasingly greater approximation in this interconnected analysis. That is, starting from a textbook, to go to a paper and from the paper back to a textbook (or viceversa). From this point of view, my analysis is only a first step (only five textbooks and a few works of twenty authors have been selected). No 19th century textbooks have been analysed nor 20th century papers on CET. Despite these limitations one of the main aspects of this work is to make explicit the relevance of the interconnections of the two parts of the flow chart. A second aspect is the distinction of four components both in the historical and in the logical part of the diagram. This distinction between regulative principles, conceptual models, mathematical structure and experiments has permitted a precise limitation of the work. In fact, the common use of the Mathematical Potential Theory and the repeatedly asserted formal equivalence between the various schools (Maxwell 1873, Planck 1887, Hertz 1892, Schwarzschild 1903, Whittaker 1960, Hesse 1961, Feynman 1962) allowed these authors to dismiss the "cruciality" of the experiments and to concentrate on the relations between regulative principles and conceptual models. At a horizontal level (temporally simultaneous) on the upper part of the flow chart, the formal correspondence of results does not imply conceptual equivalence: there is a translation of meanings. The same happens for a vertical relation: a principle of formal correspondence with previous results is established, with a modification of the conceptual framework. Thus what appears relevant is the heuristic power of the different conceptual frameworks.

NOTES

1) G. Holton (1978) p. 97 and A. French (1979) p. 271

2) G. Holton (1978) p. 99

3) M. Hesse (1974) pp. 249-55

4) G. Holton S. Brush (1973) pp. 214-220, G. Holton (1973) pp. 21-29

5) G. Buchdahl (1980) and (1981) p. 81

6) Planck (1932), Sommerfeld (1952), Pauli (1973), Feynman (1963), Landau (1970). Sommerfeld was not a Nobel prize winner.

7) Einstein, Infeld (1938) chapt. 3, para. 1.

8) Born (1962) chapt. 5, para. 1, p. 154

9) Sommerfeld (1949) p. V

10) " " pp. 35-36

11) Feynman (1963) II, 12-18

12) " " " 12-18

13) " I, 28-2 and II, 21-1

14) " " " 28-3

15) Pauli (1973) p. 2

16) Pauli (1973) pp. 2-3

17) Landau (1970) p. 122 and pp. 214-216

18) Feynman (1963) I 28-3

19) " " II, 21-2 and 3

20) " " " 15-19

21) Sommerfeld (1952) p. 2 and p. 4

22) Planck (1932) p. 19

23) Sommerfeld (1952) pp. VI-VIII and p. 11

24) Sommerfeld (1952) p. 2

25) Sommerfeld (1952) p. 2

26) Planck (1932) part. I chapt 2,pp. 12-30

27) Planck (1960) p. 42

28) Leech (1965) p. 7 and p. 16

29) Whittaker (1937) p. 38

30) Whittaker (1937) p. 44

31) Goldstein (1980) pp. 21-22

32) Goldstein (1980) p. 21, n.

33) Whittaker (1937) p.45

34) Whittaker (1960) pp. 206-7

35) Whittaker (1937) p. 45

36) Lanczos (1970) p. 31 and p. 34

37) Feynman (1963) II, 15-9, 15-23 and 17-22

38) Goldstein (1980) p. 554
39) Planck (1932) p. 1
40) " " p. V and 1-3
41) " " p. 14
42) Lanczos (1970) p. 383
43) Landau (1970) p. 65 and 94
44) Feynman (1963) II, 27-8
45) M. Hesse (1974) ch. 1
46)
Bevilacqua (1984) "H. Hertz’s Experiments and the Shift Towards Contiguous Propagation in the Early Nineties" Rivista di Storia della Scienza, n. 2, fortcoming
47) Maxwell (1873) p. X

Bibliography