A CASE STUDY:

ELECTROMAGNETIC INDUCTION

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ABSTRACT

The present paper deals with an historical and conceptual approach to electromagnetic induction. This "case study" is devoted both to university students and teachers. It belongs to a list of case studies collected in the hypermedia "Pavia Project Physics" where, in a short time, the contents of this paper together with other materials will be available in their hypertextual and multimedial dress. Our project and our efforts are directed to a new, better understanding and appreciation of scientific culture. We are interested, particularly, in clarifying concepts and theories usually used teaching physics and normally presented from textbooks. Textbooks often make different theories mix with each other. Our aim is disentangling the different theories "knotted" by textbooks and pointing out the different models underlying these different theories. This case study takes into account ideas, models, devices and theoretical problems connected to the study of electromagnetism. Special attention is payed on the problem of symmetry.

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1. OUR METHODOLOGY FOR PHYSICS TEACHING.

About electromagnetic induction, we could remind that, just ninety years ago, Einstein claimed his unsatisfaction for usual explanation of the phenomenon. He suggested even a way to work out a solution for theoretical asymmetries. Why, after ninety years, often textbooks try to cope with such asymmetries in a confused "non-einstenian" way?

Observing how textbooks talk about electromagnetism, we realize that the theories there presented are mostly Lorentz's theories. Nevertheless, this is not completely true. Textbooks explain electromagnetism following Lorentz's interpretation but, often, they overlay other interpretations. We can find some suggestions from Einstein's theory, some suggestions from Coulomb's model of "action at distance", some suggestions from Maxwell's model of "contiguous action". The troubles don't arise from the presence of different interpretations in itself but from the unsatisfactory explanation of such interpretations and from the absence of a serious comparison. Moreover, sometimes, different interpretations are alternative interpretations; it would be necessary to clarify the clash.

About these queries, it could be useful to remind Kuhn's distinction between "normal science" and "extraordinary science". Textbooks would be the best instance of normal science; on the contrary, original papers of scientists would be the best instance of extraordinary science. Although this sharp split doesn't appear completely suitable to understand science, it is a good starting-point. Thus, in our project, we have care of offering, to teachers and students, fragments of original sources or, better, fragments of primary and secondary sources. Nevertheless, this is not enough. Not only could original sources be a good instance of extraordinary science but, sometimes, even textbooks could be. Actually, textbooks written by great physicists on the same subject offer, often, truly different interpretations about it. Obviously, it is matter of advanced textbooks, namely textbooks devoted to graduate students. Thus, beside original sources, advanced textbooks could give us a better understanding of the different conceptual models underlying science. Synthetizing these remarks, we could say: we have to approach the sources and compare textbooks with sources. Unfortunatly, in science teaching, there have been a firm misunderstanding, which sounds: approaching texts suits only to humanities.

Travelling through original sources is travelling through history too. Thus, our methodology is even an historical methodology. Our research participates of both methods of physical sciences and historical sciences. Teaching physics and history of physics walk beside each other. It needs to point out that we are not interested in adding history of physics to teaching physics, as a footnote. History of physics is "inside" physics. There is not a "physical truth" but, at every time, there is a good and useful arrangement of the physical knowledge, which arises from a complex interaction among theories, models, principles, mathematical tools, devices, technology, stratified during history. We could say that the knowledge can be only an historical knowledge. Physics in itself is a beautiful present that history has brought us. Moreover, wether the knowledge is an historical knowledge, it cannot be an absolute knowledge. The knowledge is relative and comparative. Even physics teaching cannot be but relative and comparative. Following science teaching from elementary school to university, it should become more and more relative and comparative.

How compare different theories and different interpretations? Following general ideas of G. Holton and G. Buchdahl, ten years ago, F. Bevilacqua suggested a four dimensional reference frame where physical theories could be placed and compared.[1] The four components of such a reference frame are:

a) regulative principles,

b) conceptual models,

c) mathematical tools,

d) approaches to experiments.

Point (a) refers to general ideas about the laws of nature: for instance, laws of simmetry or conservation.

Point (b) refers to the manner of representing physical interactions: for instance, forces who act at distance in an empty space or actions propagating through a medium at finite speed.

Point (c) refers to the mathematical traslation of theories and, eventually, to the creation of new, suitable, mathematical tools.

Point (d) refers to the relationship between theories and experimental events related to them.

With the help of this «4-components» methodological tool, we can afford the wide research needed by a "case study".

The four components are a useful tool to make our methodological gear operate, following a strategy very close to Buchdahl's "Reduction-Realisation".[2] We could image a physical theory as a crude phenomenon worn with an interpretation. All four components participate to this interpretation. Our methodology requires two steps. In the first step, the theory is «undressed»: we remove its interpretation. At the end of this first step, we should have, in front of us, the «bare» phenomenon. In the second step, we have to identify carefully all elements which contribute to the interpretation. This analysis follows the four directions of the «4-components» methodological space. Then, component by componen, we «dress» the phenomenon with all elements of its interpretation. During this process of reconstruction, we become aware of the «meaning» of a physical theory. Following this strategy, we achieve a deeper knowledge than textbooks could give us.

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[1] BEVILACQUA F., *The Principle of Conservation of Energy and the History of Classical Electromagnetic Theory*, La Goliardica Pavese, 1983, pp. 10-34.

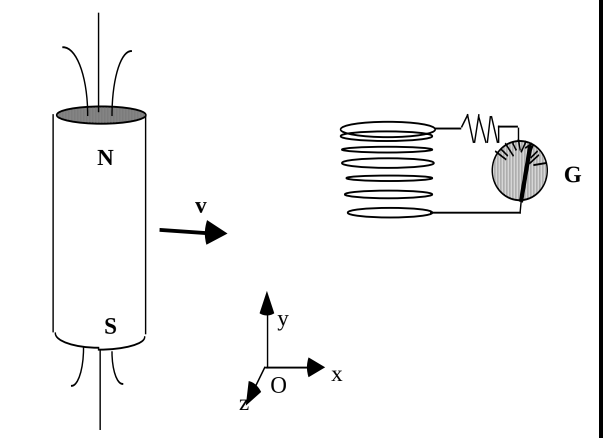
[2] Cfr. BUCHDAHL G., "Styles of Scientific Thinking", *Science & Education* 2, 1993, pp. 149-167.

If textbooks usually give one and only one explanation of each physical phenomenon - the "true" explanation - we would like, on the contrary, to present the phenomena together with their different interpretations, the different interpretations that factually occurred in the history of physics. For the case study "electromagnetic induction" we chose the interpretations of four scientists: Faraday, Maxwell, Lorentz, Einstein.

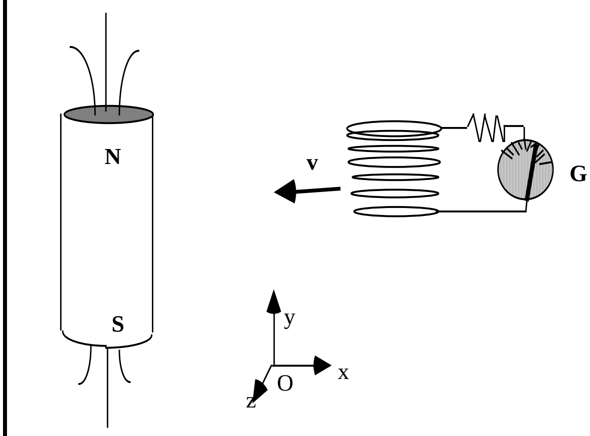
2. THEORETICAL QUIERIES.

2.a The theoretical asymmetry.

The simplest way to discover the electromagnetic induction is the relative motion between a magnet and a conducting coil. The phenomenon is symmetric with regard the change of the two reference frames: the resting conductor or the resting magnet. The induced current is the same in both cases.



The conducting coil is at rest in the reference fr
The magnet is moving in the reference frame "O
Electric current is flowing through "G".



The conducting coil is moving in the reference fr
The magnet is at rest in the reference frame "O".
The same electric current is flowing through "G'

The phenomenon depends only on the relative motion between the source of the magnetic field and the conductor.

But, usually, textbooks offer two different explanation for the two cases and two different equations

$$\boxed{\mathbf{F} = q\ \mathbf{v} \wedge \mathbf{B}} \quad (\varepsilon_M) \qquad\qquad \boxed{\nabla \wedge \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}} \quad (\varepsilon_C)$$

The first equation refers to the resting reference frame of the magnet. The second equation refers to the resting reference frame of the conducting coil. In the first equation, electromagnetic induction is produced by the magnetic force acting on the moving electric charges of the moving conducting coil. In the secon equation, the electromagnetic induction arises from the change in magnetic intensity around and through the resting conducting coil, produced by the moving magnet. Both equations should follow the "flux law": the "electromotive force" ε, induced along the coil, depends on the rate of change of the "magnetic flux" through the conducting coil.

$$\varepsilon = -\frac{d\,\Phi(\mathbf{B})}{d\,t}$$

What is the problem? On one side we have a symmetric law, the "flux law"; this law does not depend on the choice of the reference frame. It follows from Faraday's interpretation: the induced current depends on the number of "lines of force" «cut» by the conducting coil in unit time. The conducting coil could «cut» the lines of force or the lines of force could «cut» the conducting coil.

But, when we look for an explanation of the phenomenon, we find two different equations and two different forces: a magnetic force, in the resting reference frame of the magnet, an electric force in the resting reference frame of the conducting coil, according to (ε_M) and (ε_C). The same phenomenon requires two different explanations.

We can cope with this asymmetry at two different levels.

2.b Suggestions for a «mathematical» solution

At a first level we could try to unify equations (ε_M) and (ε_C) so that they could be both expressed by the "flux rule".

First of all, we realize that equation (ε_m) could be written in terms of electromotive force. Integrating along a closed path "C" of the coil, we have

$$\varepsilon = \oint_C \frac{\mathbf{F}}{q} \bullet d\mathbf{l} = \oint_C (\mathbf{v} \wedge \mathbf{B}) \bullet d\mathbf{l} \qquad (\varepsilon'_m)$$

In a similar way, integrating on the coil surface "S", (ε_C) could be written as

$$\int_S (\nabla \wedge \mathbf{E}) \bullet d\mathbf{a} = \int_S -\frac{\partial \mathbf{B}}{\partial t} \bullet d\mathbf{a}$$

ApplyingStokes' theorem, we have

$$\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \bullet d\mathbf{a} \quad \text{or} \quad \oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{\partial \Phi_B}{\partial t} \qquad (\varepsilon'_c)$$

where, once again, "C" is the closed path which sourronds the coil surface "S".

In the right hand-side of the previous equations, a partial time derivative appears. We should express it in terms of the total derivative or "convective" derivative:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \bullet \nabla$$

Thus, applying some properties of the vector calculus,

$$\frac{d\,\mathbf{B}}{dt} = \frac{\partial\,\mathbf{B}}{\partial t} + (\mathbf{v}\bullet\nabla)\mathbf{B} = \frac{\partial\,\mathbf{B}}{\partial t} + \nabla\times(\mathbf{B}\times\mathbf{v}) + \mathbf{v}(\nabla\bullet\mathbf{B})$$

where **v** does not change in time. In the right-hand side of this equation, the first term represents the change in time of **B**, the second term represents the change due to the motion of the surface "S".[3] The last term vanishes because of the known properties of **B**. Thus,

$$\frac{\partial\,\mathbf{B}}{\partial t} = \frac{d\,\mathbf{B}}{dt} - \nabla\wedge(\mathbf{B}\wedge\mathbf{v}) = \frac{d\,\mathbf{B}}{dt} + \nabla\wedge(\mathbf{v}\wedge\mathbf{B})$$

The equation (ε'_C) becomes:

$$\oint_C \mathbf{E}\bullet d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\bullet d\mathbf{a} - \int_S \nabla\wedge(\mathbf{v}\wedge\mathbf{B})\bullet d\mathbf{a}$$

or, applying Stokes' theorem,

$$-\frac{d}{dt}\int_S \mathbf{B}\bullet d\mathbf{a} = \oint_C (\mathbf{E} + \mathbf{v}\wedge\mathbf{B})\bullet d\mathbf{l}\;.$$

The left-hand side is nothing but the change in time of the magnetic flux. It follows that we could define the "electromotive force" in such a way:

$$\varepsilon = \oint_C (\mathbf{E} + \mathbf{v}\wedge\mathbf{B})\bullet dl$$

This expression accounts for both electric and magnetic contribution.[4] These contributions depend on the choice of the reference frame. In general, the phenomenon consists of the sum of the two contributions.

$$\varepsilon = \oint_C (\mathbf{E} + \mathbf{v}\wedge\mathbf{B})\bullet dl \equiv -\frac{d\,\Phi(\mathbf{B})}{d\,t}$$

2.c Suggestions for a different point of view

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[3] Cfr.: PANOFSKY W.K.H., PHILLIPS M., *Elettricità e magnetismo*, Ambrosiana, 1966, p. 165; JACKSON J.D., *Classical Electrodynamics*, J. Wiley & S., 1975, p. 212.

[4] Cfr. SCANLON P.J., HENRIKSEN R.N., ALLEN J.R, "Approaches to Electromagnetic Induction", *Am. J. Phys.*, vol. 37, n. 7, 1969, p. 701.

Nevertheless, it remains a deeper asymmetry, the asymmetry between the two different contributions, electric and magnetic. We could find out a stronger solution if we could «unify» electric and magnetic field, if we could consider electric or magnetic field as different aspects of the same field. Actually, this kind of solution was found out by Lorentz and Einstein. Lorentz and Einstein, even with important conceptual differences, offered this kind of answer. In both theories, when we change the reference frame, a «pure» electric or magnetic field transforms in a mix of electric and magnetic field.

Lorentz's theory presents this type of transformations:

$$\boxed{\mathbf{E}' = \mathbf{E} + \frac{\mathbf{v}}{c} \wedge \mathbf{B}\ ; \qquad \mathbf{B}' = \mathbf{B} - \frac{\mathbf{v}}{c} \wedge \mathbf{E}},$$

and Einstein's theory presents

$$\begin{array}{ll} \mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel} & \mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel} \\ \mathbf{E}'_{\perp} = \gamma\left[\mathbf{E}_{\perp} + (\beta \wedge \mathbf{B})_{\perp}\right] & \mathbf{B}'_{\perp} = \gamma\left[\mathbf{B}_{\perp} - (\beta \wedge \mathbf{E})_{\perp}\right] \end{array}$$

where $\beta = \mathbf{v}/c$ and $\gamma = \bullet 1\text{-}\beta^2$.

3. DIFFERENT INTERPRETATIONS THROUGH HISTORY.

At this point, after having quoted Lorentz and Einstein, would be useful to explore the different solutions to theoretical quieries. In this chapter, we will inquire into ideas and images with which four scientists represented the physical world and the physical actions. These different conceptual models corrispond to different approaches and even different interpretations of electromagnetic induction. Just now, we would like to point out the following fact: there is not a "one to one" relationship between the scientists and their conceptual models. Seldom, the original sources of each scientist offer two or more different interpretations of the same phenomenon.

We would like to present, briefly, the conceptual models of Faraday, Maxwell, Lorentz, Einstein.

3.a Faraday

Faraday analysed all different appearences of the phenomenon: change of electric current in the primary circuit, relative motion between primary and secondary circuit, relative motion between a magnet and the secondary circuit. Faraday pointed out the symmetry of the phenomenon: under the same other conditions, the same current flows through the secondary circuit either the magnet is at rest in the laboratory and the cicuit is in motion in it, or the circuit is at rest in the laboratory and the magnet is in motion in it. The phenomenon depends only on the <u>relative motion</u> between circuit and magnet.

Faraday explained the induced current as a result of the interaction betweeen the secondary circuit and the magnetic <u>lines of force</u> that are spread by the magnet or by the primary circuit. Any magnet or any circuit in which an electric current flows, spreads through space a magnetic "action" in form of lines of force. These lines correspond to the lines drawn by iron filings in the neighbourhood of a magnet.

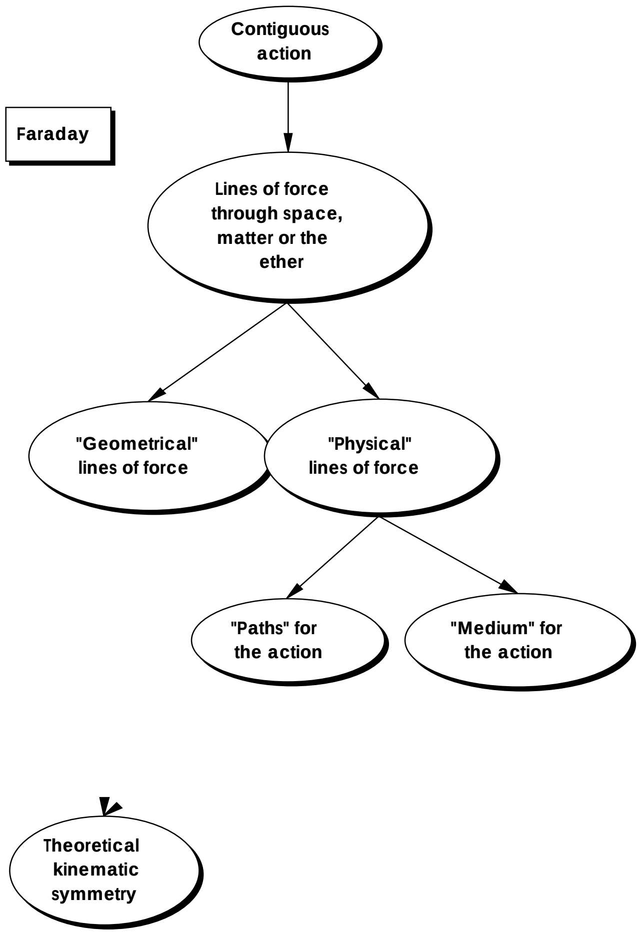
An induced current flows in a secondary circuit when the latter cross the lines of force or, conversely, when lines of force cross it. The amount of induced current depends on the number of lines of force crossed in a definite time.

Faraday refused the conceptual model of Frech physicist, belonging to the "newtonian" tradition: forces acting at distance between charged bodies or currents. On the contrary, he considered electromagnetic induction as the risult of a contiguous action propagating through space or medium. Lines of force represent the way of propagating such a action.

The Faraday’s conceptual model leaves some open queries. Faraday’s original sources offer a lot of thoughts and reflexions but not definite answers. We report here two kinds of queries.

a) Do lines of force need a medium for their propagation, or they can propagate through «pure» space? Whether a medium needs, should it be ordinary matter or ether?

b) Are lines of force a «geometrical» model or a «physical» model? Which kind of reality have them? It appears to us that sometimes Faraday preferred the first interpretations, sometimes the second one. A further query arises from Faraday's works: according to the «physical» model, are lines of force intended like «paths» or «medium» for the propagation of the contiguous action? Have lines of force material consistency in addition to physical reality?[5]



[5] Cfr. NERSESSIAN N.J., *Faraday to Einstein: Constructing Meaning in Scientific Theories*, Martinus Nijhoff Publishers, 1984, pp. 58-63, 66.

3.b Maxwell

Maxwell often stated his agreement with Faraday's conceptual model of contiguous action. In the Maxwell's *Treatise*, we find two explanations for the electromagnetic induction. First of all, he presents the Faraday's explanation in terms of <u>lines of force</u>: the induced current arises from the "cutting" between the circuit and the lines of force. Then Maxwell presents an equation for the "electromotive intensity" **E**:

$$\boxed{\mathbf{E} = \mathbf{v} \wedge \mathbf{B} - \frac{d\,\mathbf{A}}{dt} - \nabla \Psi}.$$

B is the "magnetic induction" and **A** is the "electrokinetic momentum", at present named "vector potential". In the right-hand side of the equation, the first two terms represents the two symmetric arrangement of electromagnetic induction: a conducting coil in motion through a magnetic field and a magnetic field which change in time around a conducting coil. In the firs case, the coil "cuts" the lines of force, in the second case, the lines of force "cut" the coil. Two different mathematical terms account for the two experimentally symmetric cases. The <u>symmetry has got lost</u>.

Maxwell was aware of the symmetry of the phenomenon. He tried to recover the kinematic invariance. He tried to proof that the equations for the electromotive force are invariant for a uniform motion. He acknowledged that this proof was correct only for closed currents.

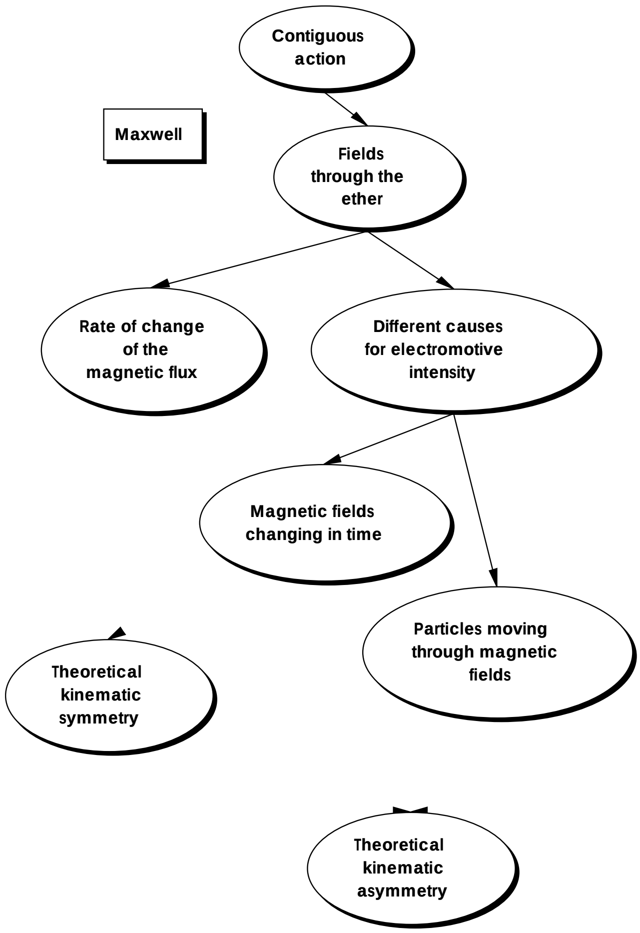
Nevertheless, the mathematical split between the two cases does not disappear. This theoretical asymmetry reflects a more general dichotomy between two conceptual models used by Maxwell: contiguous action, at one side, and forces and particles, at the other side. We suggest a reconstruction of the Maxwell's equations.[6] As we can see, they link fields to fields and fields to currents, charges, forces.

[A] Magnetic induction	$\mathbf{B} = \mathrm{rot}\,\mathbf{A}$
[B] Electromotive force	$\mathbf{E} = \mathbf{v} \wedge \mathbf{B} - \dot{\mathbf{A}} - \nabla\psi$
[C] Mechanical force	$\mathbf{F} = \mathbf{I} \wedge \mathbf{B} + e\mathbf{E} - m\nabla\Omega$
[D] Magnetization	$\mathbf{B} = \mathbf{H} + 4\bullet\mathbf{J}$
[E] Electric currents	$4\bullet\mathbf{I} = \mathrm{rot}\,\mathbf{H}$
[F] Current of conduction	$\mathbf{I}' = C\mathbf{E}$
[G] Electric displacement	$\mathbf{D} = (1/4\bullet)K\mathbf{E}$
[H] True currents	$\mathbf{I} = \mathbf{I}' + \dot{\mathbf{D}}$

[6] Cfr. EVERITT, C.W.F., "Maxwell J.C." in *Dictionary of Scientific Biography*, vol. 9, 1974.

[J] Induced magnetization	$\mathbf{B} = \mu\mathbf{H}$
[K] Electric volume density	$e = \text{div}\,\mathbf{D}$
[L] Magnetic volume density	$m = \text{div}\,\mathbf{J}$

In the Maxwell's theory, the different conceptual models appear unified by the concept of aether. The interactions among fields and between fields and their sources propagate through the aether, at finite speed. The phenomenon of electromagnetic induction too takes place through the aether. The aether is the preferred reference frame: from the theoretical point of view, it breaks whatevere kinematic symmetry.



3.c Lorentz

In Lorentz' theories, two conceptual knots found full expression:

a) the double representation of electromagnetism both in terms of field and in terms of forces and particles,

b) the problem of kinematic symmetry.

These are two general quieries, arising from the attempt of unifying mechanichs and electromagnetism. Such a general approach deals with the electromagnetic induction too. The starting point was electromagnetic phenomena should be invariant for uniform motion. From this hypotesis, "peculiar" transformation equations between inertial reference frames follow. These transformations deals with space and time but even electric and magnetic fields. From a given reference frame to another reference frame, electric and magnetic fields transform, at the first order, in such a way:

$$\mathbf{E}' = \mathbf{E} + \frac{\mathbf{v}}{c} \wedge \mathbf{B}\ ; \qquad \mathbf{B}' = \mathbf{B} - \frac{\mathbf{v}}{c} \wedge \mathbf{E}$$

where **v** is the relative speed between the two reference frames.

This theoretical approach reflects the kinematical symmetry of the electromagnetic induction phenomena.

But, from a different point of view, Lorentz points out the known theoretical asymmetry. He introduced the famous equation which links the electric and magnetic fields through the aether with the force acting on a particle because of the fields.

$$\mathbf{F} = q\,(\mathbf{D} + \mathbf{v} \wedge \mathbf{H})$$

This equation splits the force in two sides: an electric side and a magnetic side. Magnetic forces can act only on particles in motion through the aether. With regard the electromagnetic induction, we find again the already quoted asymmety:

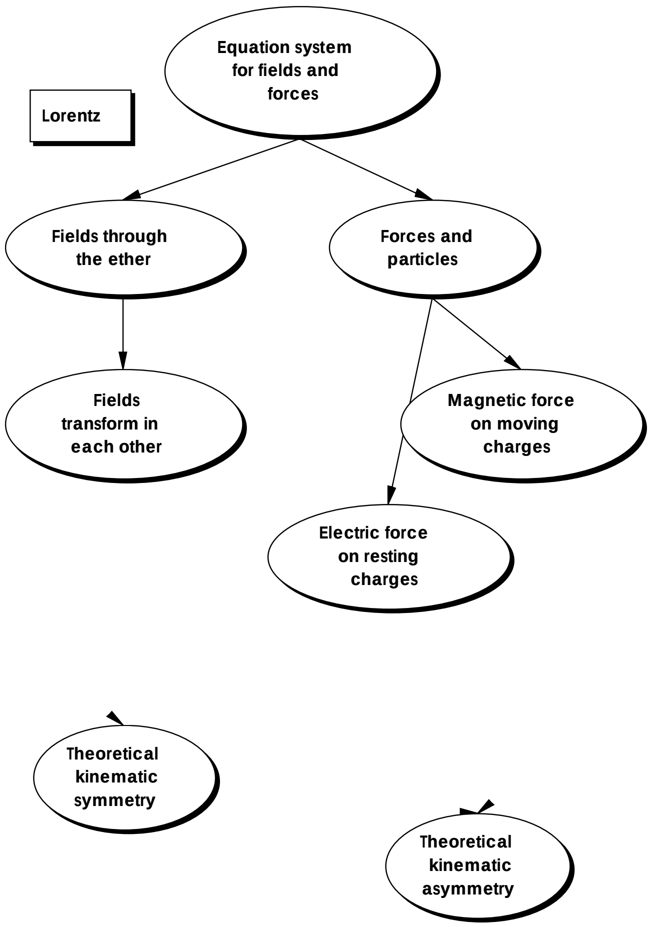
a) magnetic force acting on the conducting coil electrons,

b) electric force acting on the resting conducting coil electrons, due to the magnetic field changing in time.

The so-called Lorentz' force equation, should be added to the four equations connecting fields and their changes to electric charges and currents.

$$\mathrm{div}\,\mathbf{D} = \rho$$
$$\mathrm{div}\,\mathbf{H} = 0$$
$$\mathrm{rot}\,\mathbf{H} = 4\bullet(\mathbf{J} + \dot{\mathbf{D}})$$
$$4\bullet c^2\,\mathrm{rot}\,\mathbf{D} = -\,\dot{\mathbf{H}}$$

Altogether, the five equations should make a powerful tool to understand all electromagnetic phenomena.



3.d Einstein

Just in the introduction of his 1905 basic article "On the electrodynamics of moving bodies", we find important statements about electromagnetic induction. He claimed his unsatisfaction for the usual interpretation of "the reciprocal electrodynamic interaction of a magnet and a conductor." Einstein found unsatisfactory the asymmetry of the theory in face of the symmetry of the phenomenon. The conducting coil moving with regard the magnet was explained by a magnetic force, the magnet moving with regard the coil was explained by an electric force. He thought the two symmetric arrangement should undergo the same theoretical interpretation. Just this kind of reflections led Einstein to construct the special theory of Relativity. As you know, the theory is founded on two postulates:

I°) the principle of relativity both for mechanics and electromagnetism,

II°) the invariance of the light speed in empty space.

The contents of these postulates are similar to some Lorentz' results by the theoretical point of view is quite different. We underline two elements which keep Einstein far from Lorentz:

a) a preferred reference frame cannot exist and the aether is needless,

b) the opposite logical approach to the invariance.

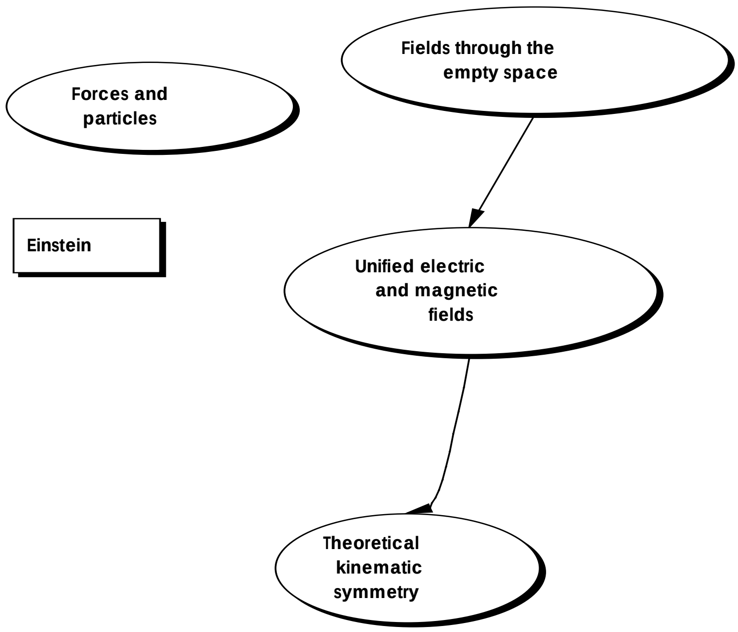
If Lorentz started from the invariance of the electromagnetic equations to reach transformation equations for space and time, Einstein started from kinematic consideration about space and time to find out transformation equations for the electric and magnetic fields. Electric and magnetic fields, from a given reference frame to another reference frame, transorm in the following way:

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel} \qquad \mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}$$
$$\mathbf{E}'_{\perp} = \gamma\left[\mathbf{E}_{\perp} + (\beta \wedge \mathbf{B})_{\perp}\right] \qquad \mathbf{B}'_{\perp} = \gamma\left[\mathbf{B}_{\perp} - (\beta \wedge \mathbf{E})_{\perp}\right]$$

where β = **v**/c and γ = •1-β2.

As far as space and time does not exist in itself but their relationship depends on the choice of the reference frame, even electric and magnetic fields do not exist in themselves. It exists an unified electromagnetic field: its electric or magnetic component depends on the choice of the reference frame. The field described by an observer as a pure electric or magnetic field, becomes a combination of electric and magnetic field for another observer.

Whatever asymmetry disappears in the interpretation of electromagnetic induction. The electric force on the conductor, arising from the moving magnet, and the magnetic force arising from the moving conductor are two aspect of the same force, an electromagnetic force.

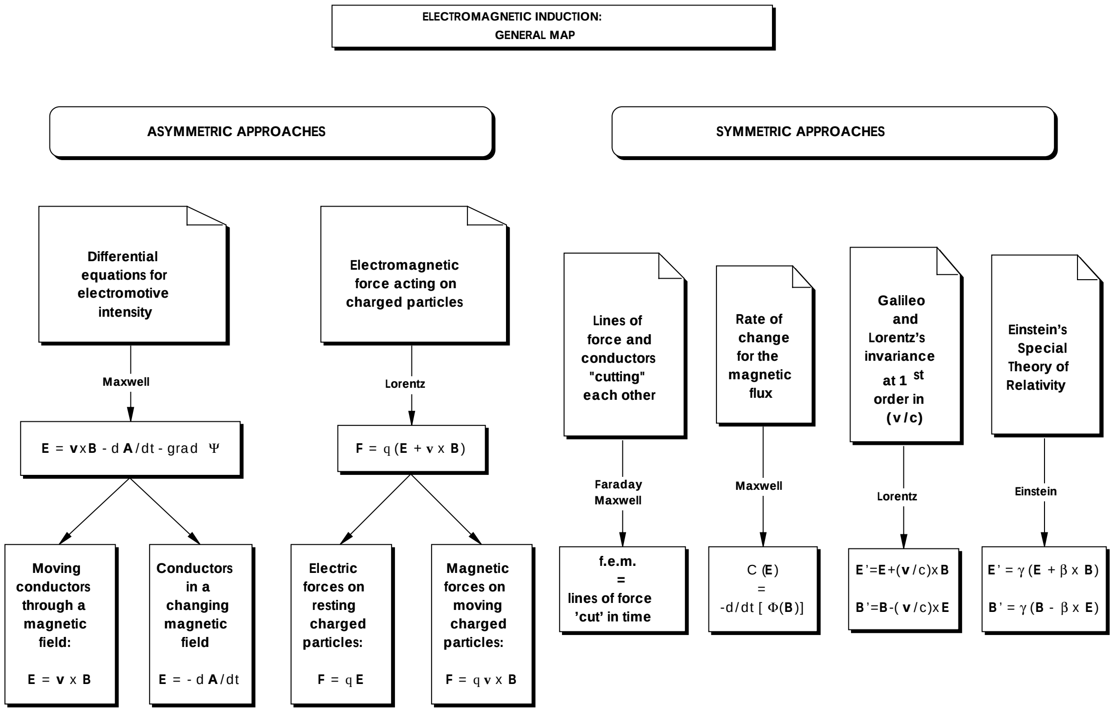


3.e Maps

Exploring Faraday's, Maxwell's, Lorentz' and Einstein's theories, should make us able to clarify all conceptual model underlying textbooks accounts. The different conceptual models operate on the electromagnetic induction phenomenon as interpretative tools. The benchmark of our inquire through different interpretations is the «key word» symmetry.

Faraday's and Einstein's theories are surely symmetric theories, Maxwell's and Lorentz' theories present both symmetric explanations and asymmetryc explanation. In Faraday, we find conductors and lines of force intersecting each others. In Einstein, we find a deep inner symmetry between electricity and magnetism. In Maxwell, we find the symmetric flux rule and the asymmetric explanation of the "causes" of induction. In Lorentz, we find symmetric transformation equations for **E** and **B** and the asymmetric account in terms of electric and magnetic forces.

At the end of this journey across both history and interpretations about electromagnetic induction, we present two maps. They show the different ways, symmetric or asymmetric, to approach the phenomenon. In this context, the word «symmetry» is a key-word: it represents even the more meaningful "regulative principle" in order to frame all theories.



4. HISTORY IN FACE OF DEVICES AND THEORIES.

This chapter deals with two devices and their operation. It is important to point out that is not our analysis only an abstract exercise. With the help of our historical and conceptual researches, we can afford even a serious understanding of technology. After getting a look to different conceptual model and different interpretations about electromagnetic induction, we can cope with some theoretical and technical quieries. Our conceptual journey through history will help us to better understand the possible solutions.

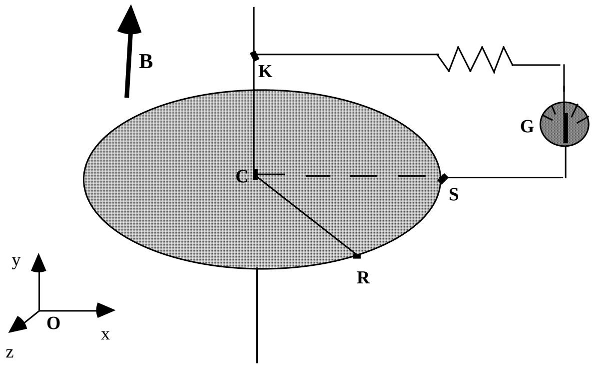
What is the main trouble? In some geometrical and kinematic arrangements of magnets, conductors and magnetic fields, the theory seems to fail. There is an interesting fact: such arrangements corrispond to technical devices that have worked well since the previous century.

4.a The homopolar inductor

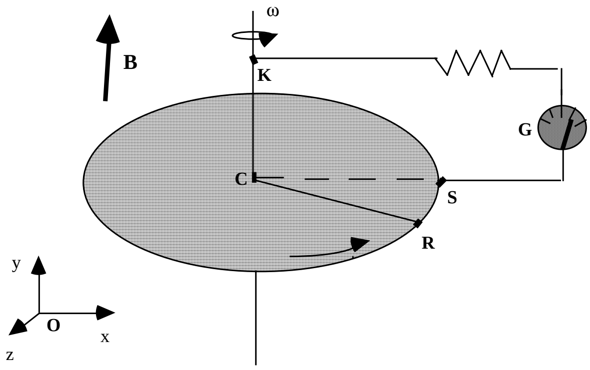
The first device and the inquiry about it are quite simple. The "homopolar inductor" consists of a conducting disk, revolving on its own axe in a magnetic field: an electric current is produced. There is the motion of the electric charges in the magnetic field, but there isn't any change in the geometry of the device; how can the electric current be produced? In such conditions, it seems could not be any change in the magnetic flux through the circuit .[7] But, as we will soon see, this is not the case.

Let's consider a conducting disk, revolving at the angular velocity ω with regard a given reference frame 'O', which is the site of a uniform magnetic field **B**.

[7] Cfr. Feynman,Leighton, Sands, *The Feynman Lectures on Physics*, Inter European Editions, 1975, vol II, part I, 17-3, 4.



The disk is at rest in the reference frame "O"



The disk is moving in the reference frame "O"

If we choose the closed line CRSGKC as integration path, the radius r = CR revolves together with the disk. The electromotive force is

$$\varepsilon = \oint (\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \bullet d\mathbf{l} = \oint (\mathbf{v} \wedge \mathbf{B}) \bullet d\mathbf{l} = \\ = \int_0^r \omega x B \;\; dx = \frac{1}{2} \omega r^2 B$$

The only contribution to the magnetic flux is given by the round sector CRS. Because of the motion of the radius CR, the magnetic flux decreases during the time. Its change is

$$d\,\Phi(\mathbf{B}) = \int \mathbf{B} \bullet d\mathbf{a} = B \int d\mathbf{a} = \\ = B\left(-\frac{1}{2}\, r\;\, v\;\, dt\right) = -\frac{1}{2} \omega r^2 B\, dt$$

$$-\frac{d\,\Phi(\mathbf{B})}{dt} = \frac{1}{2} \omega r^2 B \quad .$$

The previous equation follows the general induction law ($\mathcal{E}_T$). No exception to the flux rule appears with the choice of the integration path CRSGKC. Nevertheless, if we chose a different path, we could find strange results. Let's choose the closed circuit CSGKC: the radius CS is at rest with regard the reference frame 'O'. Through the surface enclosed by the circuit the magnetic flux vanishes. Even the electromotive force vanishes, because **E** = 0, **v** = 0, **B** = 0. The general induction law ($\mathcal{E}_T$) is valid again but the electromotive force cannot vanish: the galvanometer shows an electric current flowing through the circuit.

The solution consists of a correct choice of the integration path: CKGSRC instead of CKGSC.[8] Only through the surface included in the first path there is a change of the magnetic flux. Actually, when the conductors are large and only some sides of the «circuit» are in motion, the choice of the integration path is not trivial.

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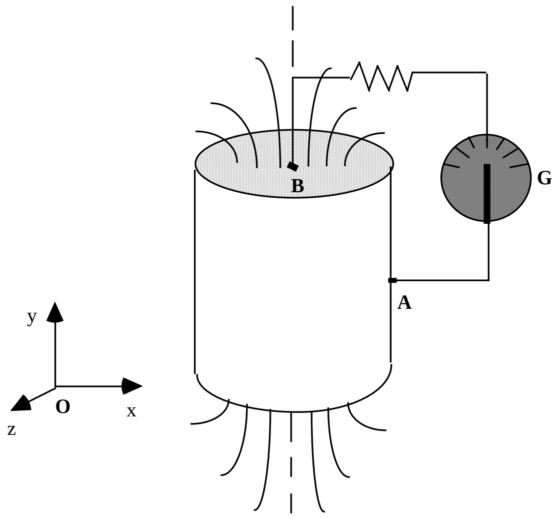
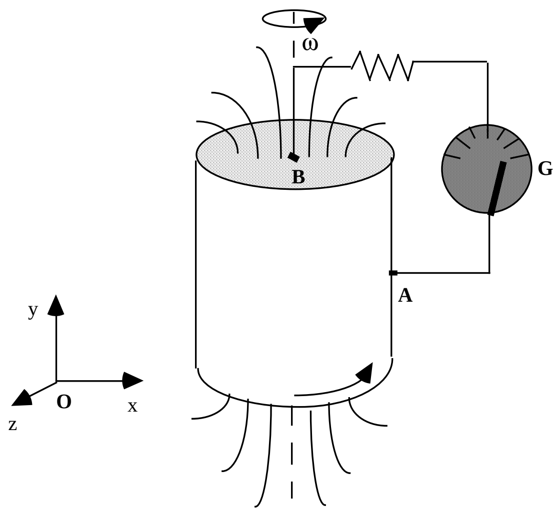
[8] Cfr.: SCANLON P.J., HENRIKSEN R.N., ALLEN J.R, "Approaches to Electromagnetic Induction", *Am. J. Phys.*, vol. 37, n. 7, 1969, p. 705.

4.b The unipolar inductor

The second device requires a more demanding inquiry. The "unipolar inductor" consists of an iron cylinder magnetized along its own axe; when it revolves on the same axe, a current arises in a conducting wire placed between a pole and the lateral curved surface. The same current arises even though the magnet is at rest and the wire revolves around it. The induced current depends only on the relative motion between the magnet and the circuit.



The magnet is at rest in the reference frame "O"



The magnet is revolving in the reference frame "O"

Two different explanations of the phenomenon work out from two different hypotheses:

a) magnetic lines rotate together with the magnet,

b) magnetic lines don't rotate together with the magnet.

Following the hypothesis (a), the moving circuit «cut» the magnetic lines of force of the resting magnet; conversely, the moving lines of force of the moving magnet «cut» the resting circuit.

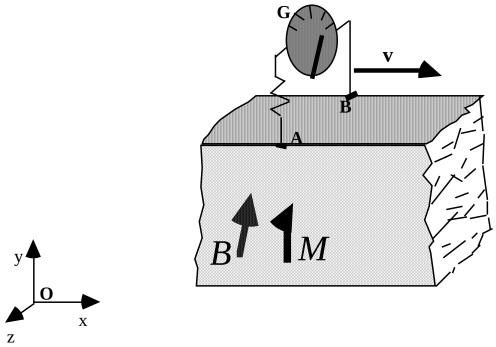
Following the hypothesis (b), the moving circuit «cut» the magnetic lines of the resting magnet, but the moving magnet revolves around its own lines and the latter can't «cut» the circuit. The current could be explaned only by an electric field arising from the revolution of the magnet.[9]

From the historical point of view, the two hypotesis are particularly interesting. In 1903, Eichenwald realized that moving electrically polarized dielectrics produced a magnetic field. The fact agreed with

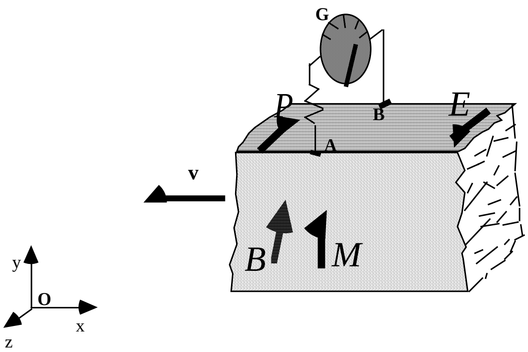
[9] Cfr. MILLER A.I., *Albert Einstein's special theory of relativity*, Addison-Wesley Pub. Com., 1981, p. 159.

Lorentz' theories but not with Hertz' theories. Lorentz himself claimed that, conversely, a moving magnet could become electrically polarized.[10]

To better analyze the electrical polarization of a magnet, let's consider a magnetized iron bar, moving along the 'x' axe and magnetized in the 'z' direction. We should suppose the 'x' length of the bar is far greater than 'y' and 'z' dimensions. A galvanometer 'G' is connected to the bar by two grazing contacts 'a' and 'b': this is an electric circuit.



The magnetized bar is at rest in the reference frame "O"
The circuit "AGB" is in motion in the reference frame "O"



The magnetized bar is moving in the reference frame "O"
The circuit "AGB" is at rest in the reference frame "O"

When the magnet is at rest and the circuit is moving along the 'x' axe with regard a given reference frame 'O', a magnetic force q**v**x**B** acts on the electrons 'q' belonging to the circuit. Because of the symmetry of the phenomenon, when the circuit is at rest and the magnet is moving along the negative 'x' axe of the same reference frame, on the electrons will act the same amount of force. The latter force could not be named «magnetic», because the electrons are at rest. It should be named «electric» force and would arise from the motion of the magnetic lines of the bar.

But, unfortunatly, this interpretation suffers some troubles. We could consider the lines of force linked to the bar and its motion. Nevertheless, the magnetic field of the bar is a steady field: at a given place, its amount does not change during the time. Then, even though we observe carefully the lines of force, we are not able to state wether the magnet is at rest or not. A moving bar produces the same magnetic field than a resting bar. We are not sure wether the bar is moving or is at rest and we are not sure wether the lines of force are moving or are at rest: no difference appears between the two arrangements. Physics cannot consider them as two different cases. The theoretical alternative between the two statements,

"a) the magnetic lines of force are moving together with the magnet",

"b) the magnetic lines of force are not moving together with the magnet",

makes not physical sense.

The Einstein's special theory of relativity offers a satisfactory theoretical point of view. The electric force acting on the circuit at rest does not arise from the motion of the lines of force. It is an «electrostatic» force arising from the electric polarization of the moving magnet.[11] The theory states

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[10] Ibidem. Thus, it seems that the hypotesis of electrically polarization of a moving magnet did not arise firstly from Einstein's special theory of Relativity. Cfr. LORENTZ H.A., "Remarque au sujet d'induction unipolaire", *Arch. Neer.*, 1904, 9, 380; reprinted in LORENTZ H.A., *Collected Papers 3*, pp. 177-179.

[11] BECKER R., *Teoria della elettricità*, Sansoni, 1949, II vol., p. 404.

that an only electrically polarized body in its own resting reference frame becomes even magnetized in another reference frame moving with regard the previous frame. Symmetrically, an only magnetically polarized body in its own resting reference frame becomes even electrically polarized in another reference frame moving with regard the previous frame.[12]

If $\mathbf{P_0}$ is the 'electric polarization' vector and $\mathbf{M_0}$ is the 'magnetic polarization' vector in the resting reference frame of a body, and **P, M** are the correspondig vectors in a reference frame moving with regard the previous frame, we can write the following equations:

$$\mathbf{M} = \frac{1}{c}\,\mathbf{P_0} \wedge \mathbf{v} \qquad\qquad \mathbf{P} = \frac{1}{c}\,\mathbf{v} \wedge \mathbf{M_0}$$

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[12] Ibidem, II, p. 398.